~ a short overview ~

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References: 1003.3483, 1011.4705, 1101.4049.

SPINFOAM COSMOLOGY

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Laboratoire de Mathématiques J.A. Dieudonné - Nice

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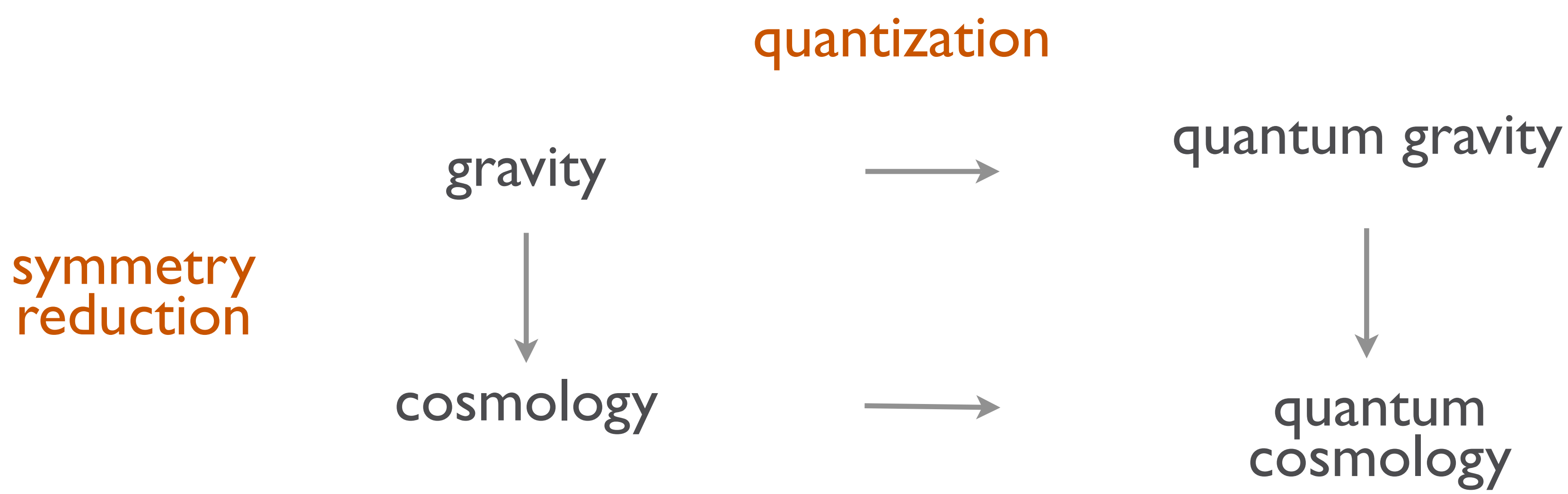
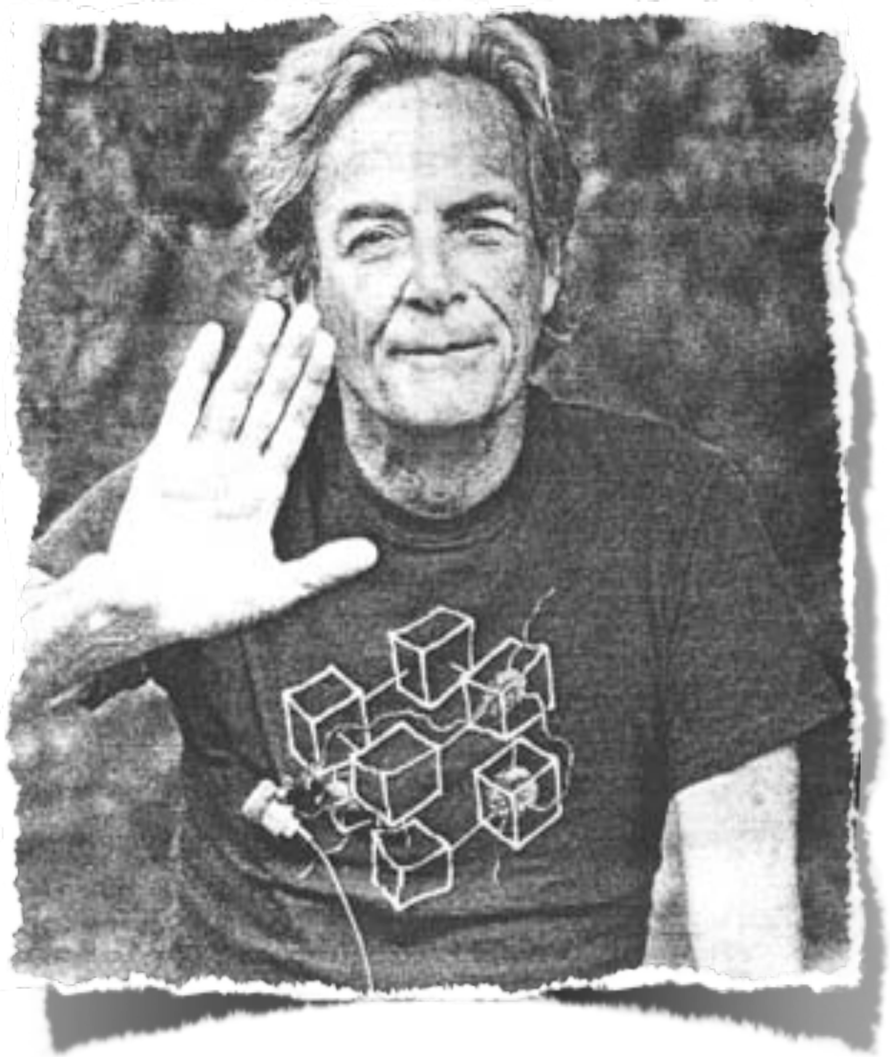
LOOP QUANTUM GRAVITY & COSMOLOGY



- How cosmology can be obtained from the full quantum gravity theory?
- RESULTS
  - There are approximations in the quantum theory that yield cosmology.
  - The theory recover general relativity in the semiclassical limit, also for non-trivial solutions.
  - There is a simple way to add the cosmological constant to the dynamics of LQG.

Han & Fairbairn, Meusburger '11

PLAN OF THE TALK



1. Definition of the complete theory
   - with the cosmological constant
2. Approximations
   - **graph**
   - **vertex**
   - **spin**
3. Study of the semiclassical limit (de Sitter solution)
   - transition amplitude
   - Hamiltonian constraint

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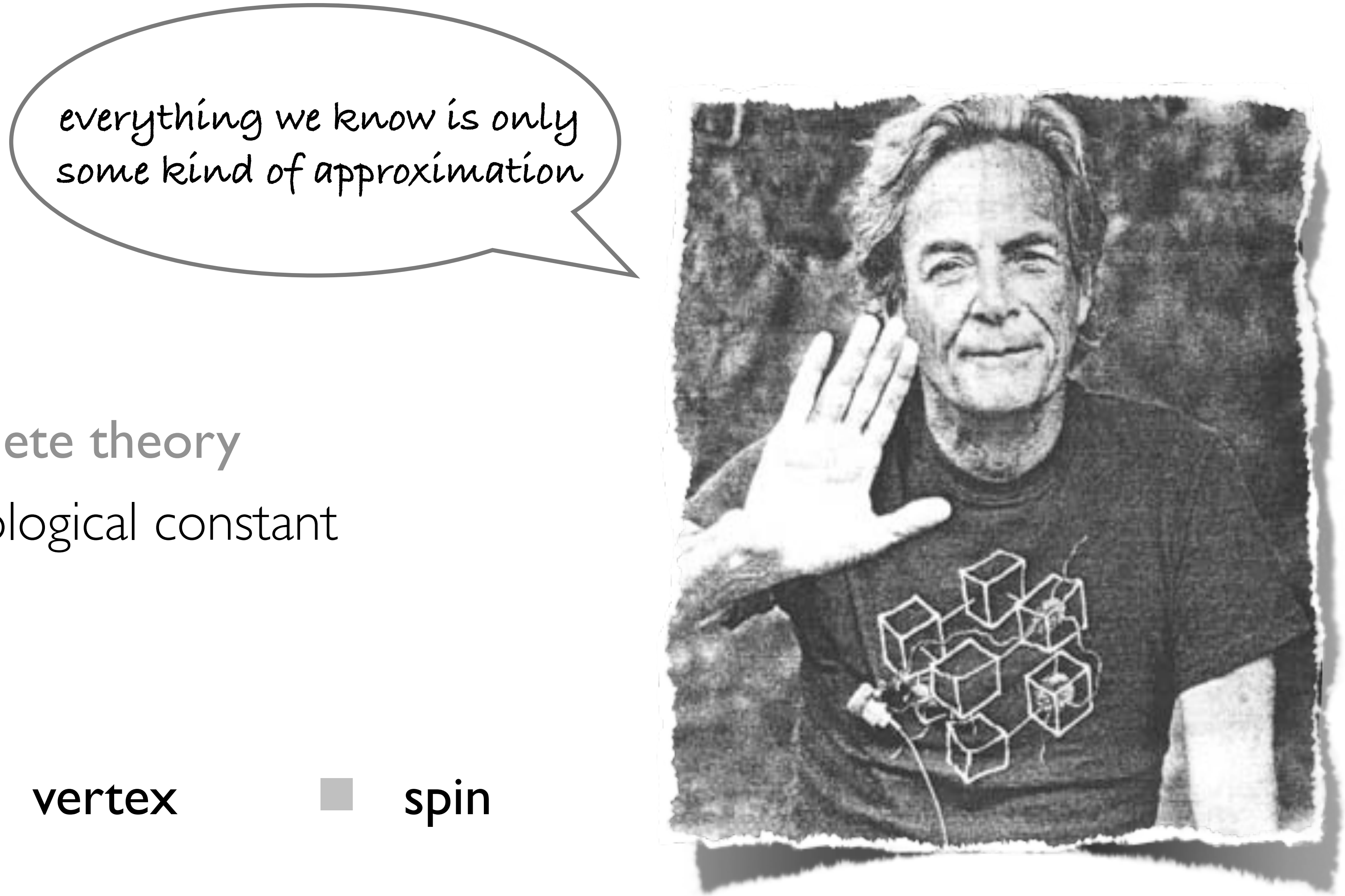

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KINEMATICS

Hilbert space: $\tilde{\mathcal{H}} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma$ where $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$

Abstract graphs: Γ is determined by N = # nodes, L = # links and their adjacency

Identifications: $\tilde{\mathcal{H}}/\sim$

- if Γ is a subgraph of Γ' then we must identify $\mathcal{H}_\Gamma$ with a subspace of $\mathcal{H}_{\Gamma'}$
- divide $\mathcal{H}_\Gamma$ by the action of the discrete group of the automorphisms of Γ

Operators: h_f are diagonal and E_f are the left-invariant vector fields

States that solve gauge constraint: $|\Gamma, j_\ell, v_n\rangle \in \tilde{\mathcal{H}} = \bigoplus_{\Gamma} \bigoplus_{j_\ell} \bigotimes_{n} \mathcal{H}_n$

COHERENT STATES

Bianchi, Magliaro, Perini '10

$$\psi_{H_\ell}(U_\ell) = \int_{SU(2)^N} dg_n \quad \prod_{l\in\Gamma} K_t(\, g_{s(\ell)}\, U_\ell\, g_{t(\ell)}^{-1}\; H_\ell^{-1})$$

"group average" to get gauge invariant states

The heat kernel K_t peaks each U_ℓ on H_ℓ

$$K_t(U) = \sum_j (2j+1)\; e^{-2t\hbar\, j(j+1)}\;\mathrm{Tr}\,[D^j(U)]$$

$$H = D^{\frac{1}{2}}(R_{\vec{n}})\; e^{-iz\frac{\sigma_3}{2}}\; D^{\frac{1}{2}}(R_{\vec{n}'}^{-1}) \qquad\qquad H_\ell \in SL(2,\mathbb{C})$$

$$z = \xi + i\eta$$

- Geometrical interpretation for the $(\vec{n}, \vec{n}', \xi, \eta)$ labels: Freidel, Speziale '10

  $\vec{n}, \vec{n}'$ are the 3d normals to the faces of the cellular decomposition;

  ξ $\leftrightarrow$ extrinsic curvature at the faces and η $\leftrightarrow$ area of the face.

- Superposition of spinnetwork states, but peaked on a given geometry.

DYNAMICS

Bianchi, Rovelli, FV '10
Bianchi, Krajeski, Rovelli, FV '11

term that yields the
cosmological constant
↓

Transition amplitude:

boundary state $\psi \in \mathcal{H}$

$$Z_{\mathcal{C}} = \sum_{j_f, \mathrm{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda\, \mathrm{v}_e} \prod_v A_v(j_f, \mathrm{v}_e)$$

Vertex amplitude:

$$A_v(j_f, \mathrm{v}_e) \longrightarrow W_v(H_\ell) = \langle A | \psi_{H_\ell} \rangle$$

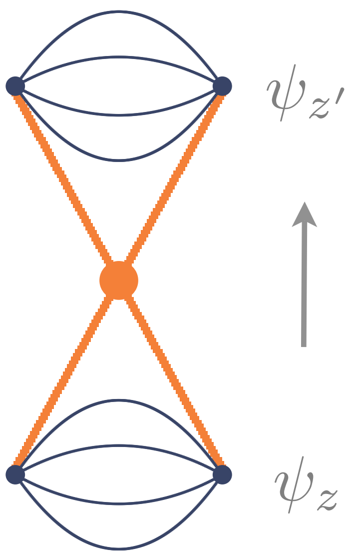
$$W_v(H_\ell) = \int_{SL(2,\mathbb{C})^N} dG_n \prod_\ell P_t(H_\ell\,,\, G_{s(\ell)} G^{-1}_{t(\ell)})$$

where

$$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)}\ \mathrm{Tr}\left[D^{(j)}(H) Y^\dagger D^{(j^+,j^-)}(G) Y\right]$$

Ingredients to do COSMOLOGY

- GRAPH TRUNCATION ↔ number of d.o.f we want to describe
  - *example:* 2 tetrahedra glued along all their faces = triangulated **3-sphere**
- GEOMETRY ↔ coherent states can peaked on a given geometry
  - we choose an **homogeneous and isotropic** geometry
    - $z_\ell \rightarrow z$ where $Re(z) \sim \dot{a}$ and $\sqrt{Im(z)} \sim a$
    - transition amplitude from an initial to a final state (boundary states are fixed)



- VERTEX EXPANSION
  - we consider the 1st order ↔ single vertex
- SEMICLASSICALITY ↔ coherent states + large distance
  - large distance $\Rightarrow$ large spin j (the graph truncation is well defined)

Evaluation of the amplitude

$$W(z) = \sum_j (2j+1)\, \frac{N_o}{j^3}\, e^{-2t\hbar j(j+1) - izj - i\lambda \mathrm{v}_o j^{\frac{3}{2}}}$$ ~ gaussian sum

$$j \sim j_o + \delta j$$

- max(real part of the exponent) gives where the gaussian is peaked;

$$j_o = \frac{Im(z)}{4t\hbar}$$

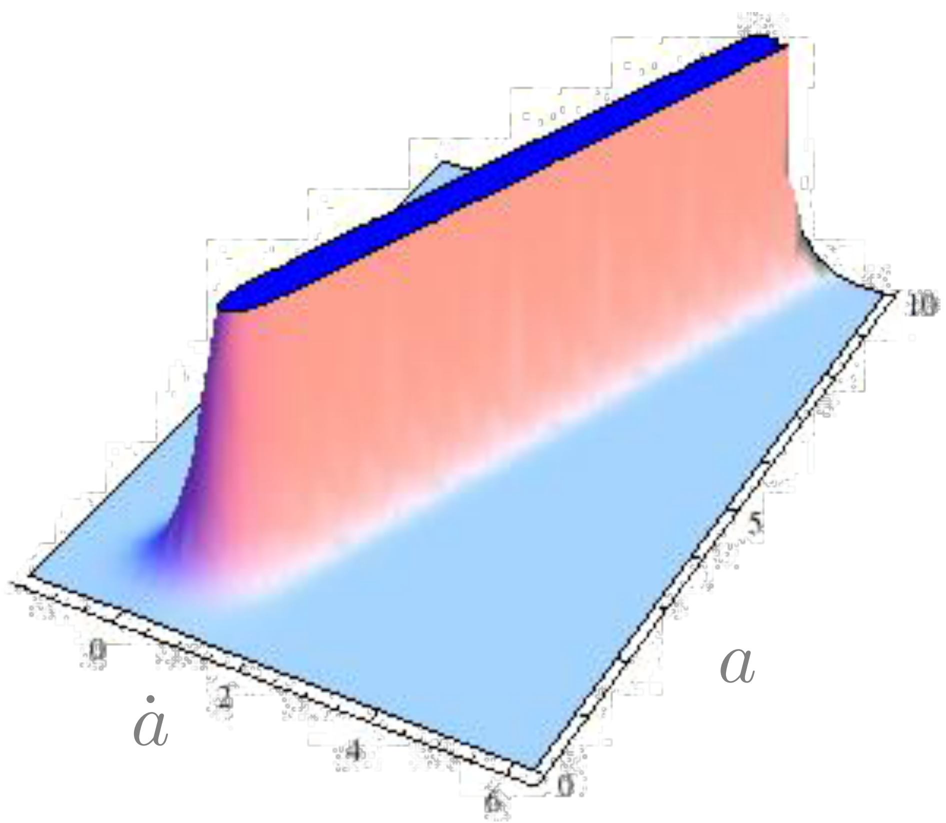
- imaginary part of the exponent =2kπ gives where the gaussian is not suppressed.

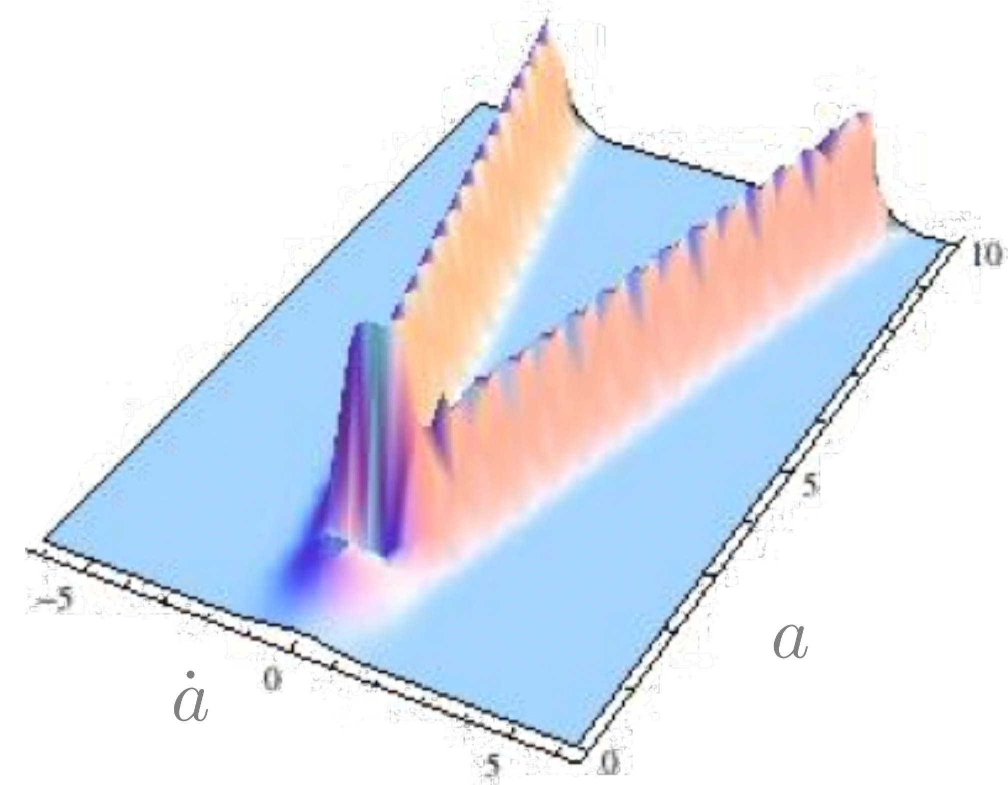
$$Re(z) + \lambda \mathrm{v}_o j^{\frac{1}{2}} = 0.$$

- $Re(z) \sim \dot{a}$ and $\sqrt{Im(z)} \sim a$

- $$\frac{Re(z)^2}{Im(z)} = \frac{\lambda^2 \mathrm{v}_o^2}{4t\hbar} \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3}$$ with $\Lambda = const\, \lambda^2 G^2 \hbar^2$

NUMERICAL EVALUATION





- $\dot{a} \propto a$
  - The numerical study confirms the validity of the approximations taken to perform the previous calculation. The extrinsic curvature is linear in the scale factor, as characteristic in de Sitter space.

- **The sign of the λ-term.**
Expanding or contracting solutions have different signs. The quantum gravity vertex (with or without cosmological constant) is the sum of two terms with opposite time orientation. Thus the final amplitude does not depend of the choice of the sign.

FROM COVARIANT TO CANONICAL

Our amplitude happens to satisfy an equation

Bianchi, Rovelli, FV '10
Bianchi, Krajeski, Rovelli, FV '11
Röken '11

$$\hat{H}(z, \frac{d}{dz})W(z', z) = \left(\hat{z}^2 - \hat{\bar{z}}^2 - \cancel{3t\hbar}\right) W(z', z) = 0$$

With the cosmological constant:

$$(z + \frac{3}{2}\lambda \mathrm{v}_o j_o^{\frac{1}{2}})^2 - \overline{(z + \frac{3}{2}\lambda \mathrm{v}_o j_o^{\frac{1}{2}})^2} = 0$$

$$i4\, Im(z)\ (Re(z) + \frac{3}{2}\lambda \mathrm{v}_o j_o^{\frac{1}{2}}) = 0$$

- $a^3 \sim 2\mathrm{v}_o j_o^{\frac{3}{2}}$

$$\frac{Re(z)^2}{Im(z)} = \frac{\lambda^2 \mathrm{v}_o^2}{4t\hbar} \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3}$$

It is possible to translate the information from the transition amplitude to an
EFFECTIVE HAMILTONIAN CONSTRAINT

FROM CANONICAL TO COVARIANT

- Idea: define a **path integral for general covariant system**. The situation is different wrt the usual path integral, because there is not a preferred choice of time.

Ashtekar, Campiglia, Henderson '09

Henderson, Rovelli, FV, Wilson-Ewing '10

- Final goal: compute transition amplitude between two state of the geometry, and **connect with spinfoam** theory.

$$\langle \vec{\lambda}_F | \vec{\lambda}_0 \rangle_{\pm\delta} = \frac{i}{2\pi} \sum_{M=0}^{\infty} \sum_{\vec{\lambda}_1 \ldots \vec{\lambda}_{M-1}} \prod_f A_f(\vec{\lambda}_f) \prod_v A_v(\vec{\lambda}_f)$$

- sum over two-complexes ↔ sum over # of transitions
  sum over colorings ↔ sum over the λ_i
  spinfoam vertices ↔ transitions
  spinfoam faces ↔ sequences of steps without transitions

To built a vertex we ask for
1. Locality
2. H should be a density
3. Lorentz invariance

- Caveat: deparametrized theory vs group averaging.
- Technical issue: **regularization** ⟶ Physical issue: the resulting **expansion is local**.
- The choice of the regularization is **not trivial**! Only regularizing the **sectors with positive and negative group averaging parameter** separately and taking into account **both sectors** we obtain an expansion where the amplitude of one history is the product of amplitudes of the elements forming the history.

SUMARY & RESULTS

- A simple way to add the cosmological constant to the dynamics of LQG.
- It is possible to compute quantum transition amplitudes explicitly in suitable approximations:
  - graph expansion
  - vertex expansion
  - large volume expansion
- There are approximations in the quantum theory that yield cosmology.
- The transition amplitude computed appears to give the correct Friedmann dynamics with Λ in the classical limit.
- The theory recovers general relativity in the semiclassical limit, also for non-trivial solutions (de Sitter space).

The cosmological principle

- The dynamics of a homogeneous and isotropic space approximates well the observed universe.
- The effect of the inhomogeneities on the dynamics of its largest scale, described by the scale factor, can be neglected in a first approximation.
- This is not a large scale approximation, because it is supposed to remain valid when the universe was small! **It is an expansion in $N \sim a/\lambda$!**
- The full theory may be recovered by adding degrees of freedom one by one, starting from the cosmological ones.
- We can define an approximated dynamics of the universe for a finite number of degrees of freedom .

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Graph Expansion

Mode expansion $\longleftrightarrow$ truncation on a graph

- **Restrict the states to a fixed graph with a finite number N of nodes.**
  This defines an approximated kinematics of the universe,
  inhomogeneous but truncated at a finite number of cells.

- **The graph captures the large scale d.o.f. obtained averaging the metric**
  over the faces of a cellular decomposition formed by N cells.

- **The full theory can be regarded as an expansion for growing N.**
  FRW cosmology corresponds to the lower order where there is only
  a regular cellular decomposition: the only d.o.f. is given by the volume.

- **Coherent states $|H_\ell\rangle$ describing a homogeneous and isotropic geometry:**

$$z = \xi_\ell + i\eta_\ell \quad \longrightarrow \quad z = \alpha c + i\beta p \qquad \forall \ell$$

  Geometry is determined by (c, p) in the past and (c′, p′) in the future.

Evaluation of the amplitude

(8) $$\langle W|\psi_{H_{(z,z')}}\rangle = W(z,z') = W(z)\,\overline{W(z')}$$

(9) $$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)}\ \mathrm{Tr}\left[D^{(j)}(H) Y^\dagger D^{(j^+,j^-)}(G) Y\right]$$

(10) $$H = D^{\frac{1}{2}}(R_{\vec{n}})\ e^{-iz\frac{\sigma_3}{2}}\ D^{\frac{1}{2}}(R^{-1}_{\vec{n}'}) \qquad H_\ell \in SL(2,\mathbb{C})$$

(11) $$D^{(j)}(H) = D^{(j)}(R_{\vec{n}})\ D^{(j)}(e^{-iz\frac{\sigma_3}{2}})\ D^{(j)}(R^{-1}_{\vec{n}'})$$

(12) $$\eta \gg 1 \qquad D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) \approx e^{-izj}\, P$$

projection on the highest magnetic number

(13)

(14) $$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)} e^{-izj}\ \mathrm{Tr}\left[P\ Y^\dagger D^{(j^+,j^-)}(G) Y\right]$$

$$W(z) = \sum_j (2j+1)\,\frac{N_o}{j^3}\ e^{-2t\hbar j(j+1) - izj - i\lambda \mathrm{v}_o j^{\frac{3}{2}}}$$

intertwiner $\mathrm{v}_e \sim \mathrm{v}_o\, j^{3/2}$