Spinfoam Cosmology

Francesca Vidotto

Centre de Physique Théorique, Marseille

INFN & DFNT, Pavia

Spinfoam Cosmology

quantum cosmology from the full theory

Francesca Vidotto

Centre de Physique Théorique, Marseille

INFN & DFNT, Pavia

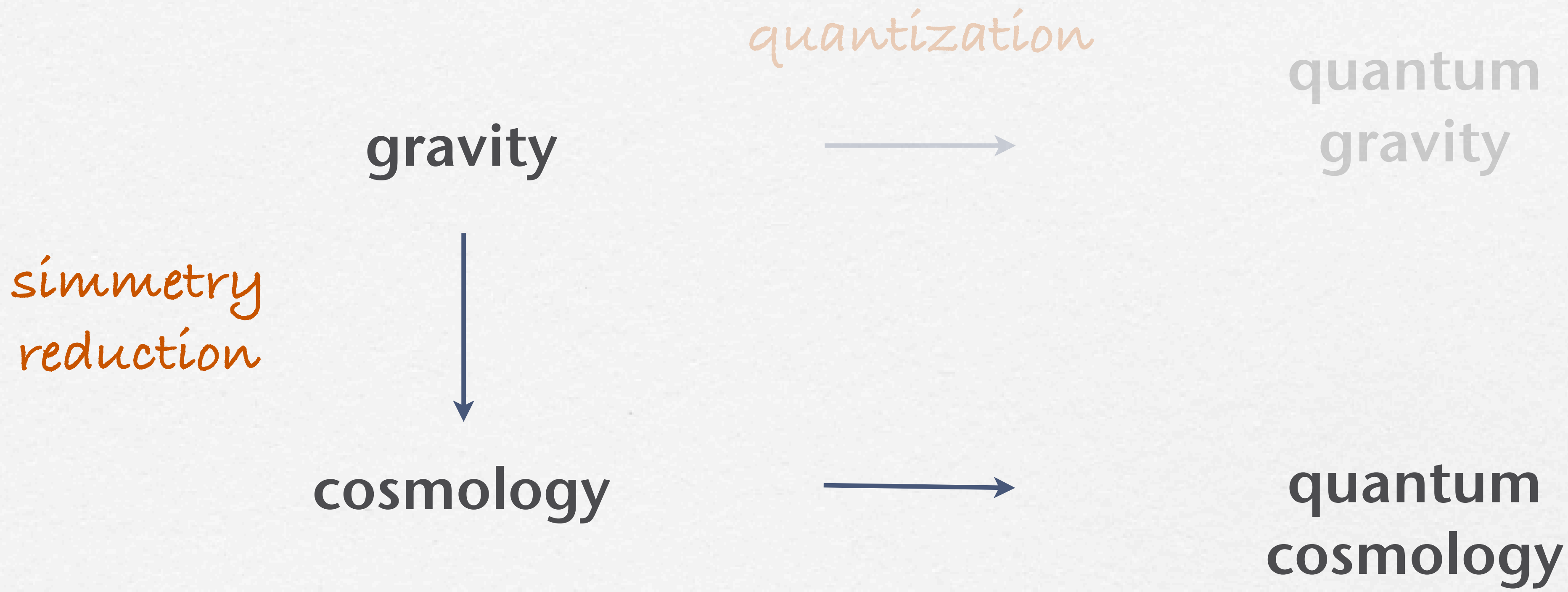
What is quantum cosmology?

gravity

What is quantum cosmology?

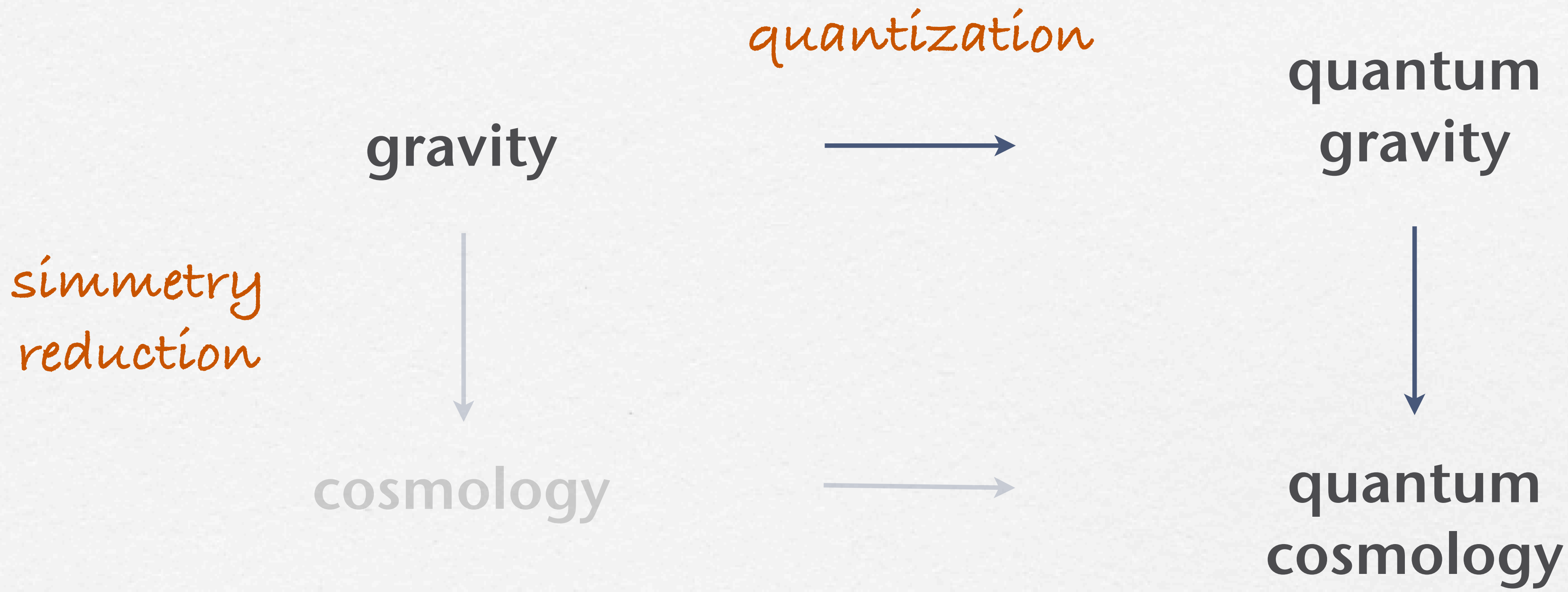
gravity → *quantization* → quantum gravity

What is quantum cosmology?

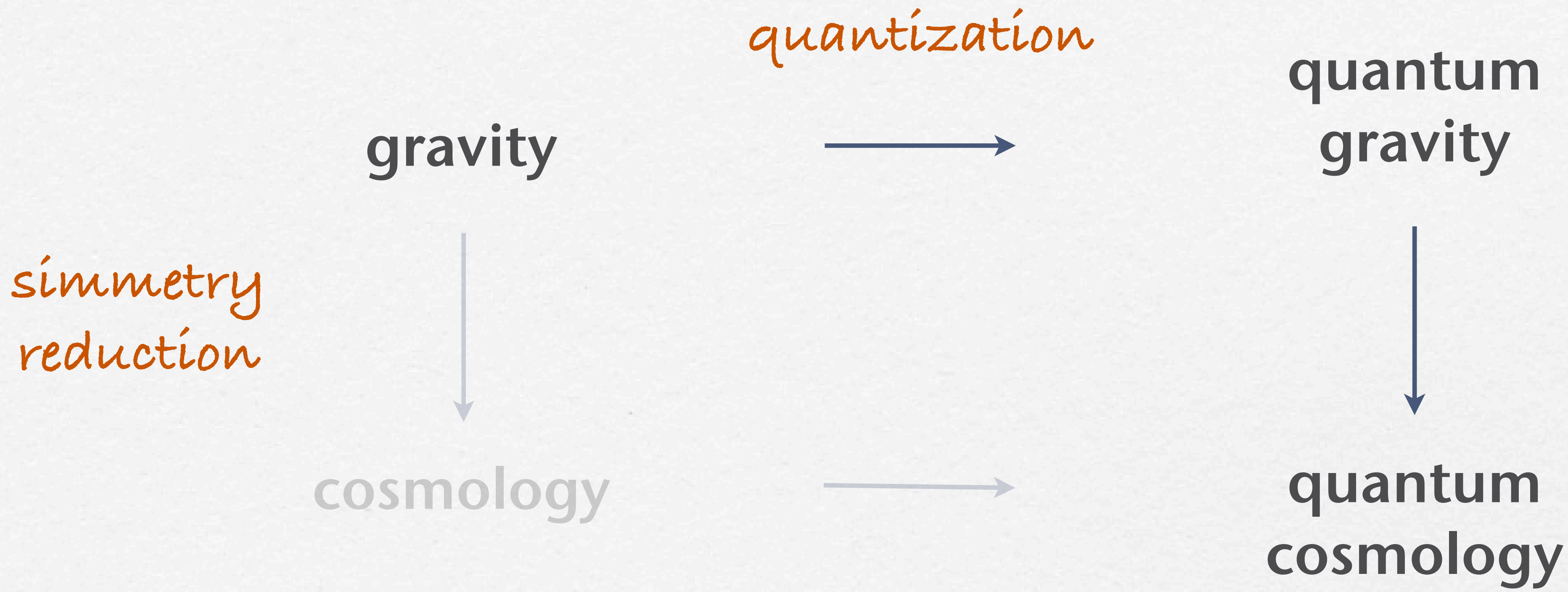


- Wheeler, DeWitt, Misner (1967)
- LOOP QUANTUM COSMOLOGY
  Bojowald (1999), *see the talks by Mena-Marugan, Tanaka, Martín-Benito, Olmedo...*

What is quantum cosmology?



What is quantum cosmology?



- ☐ What is the relation between LQC and full LQG?
- ☐ Can we describe the full quantum geometry at the bounce?
- ☐ Can we include "naturally" inhomogeneities ?

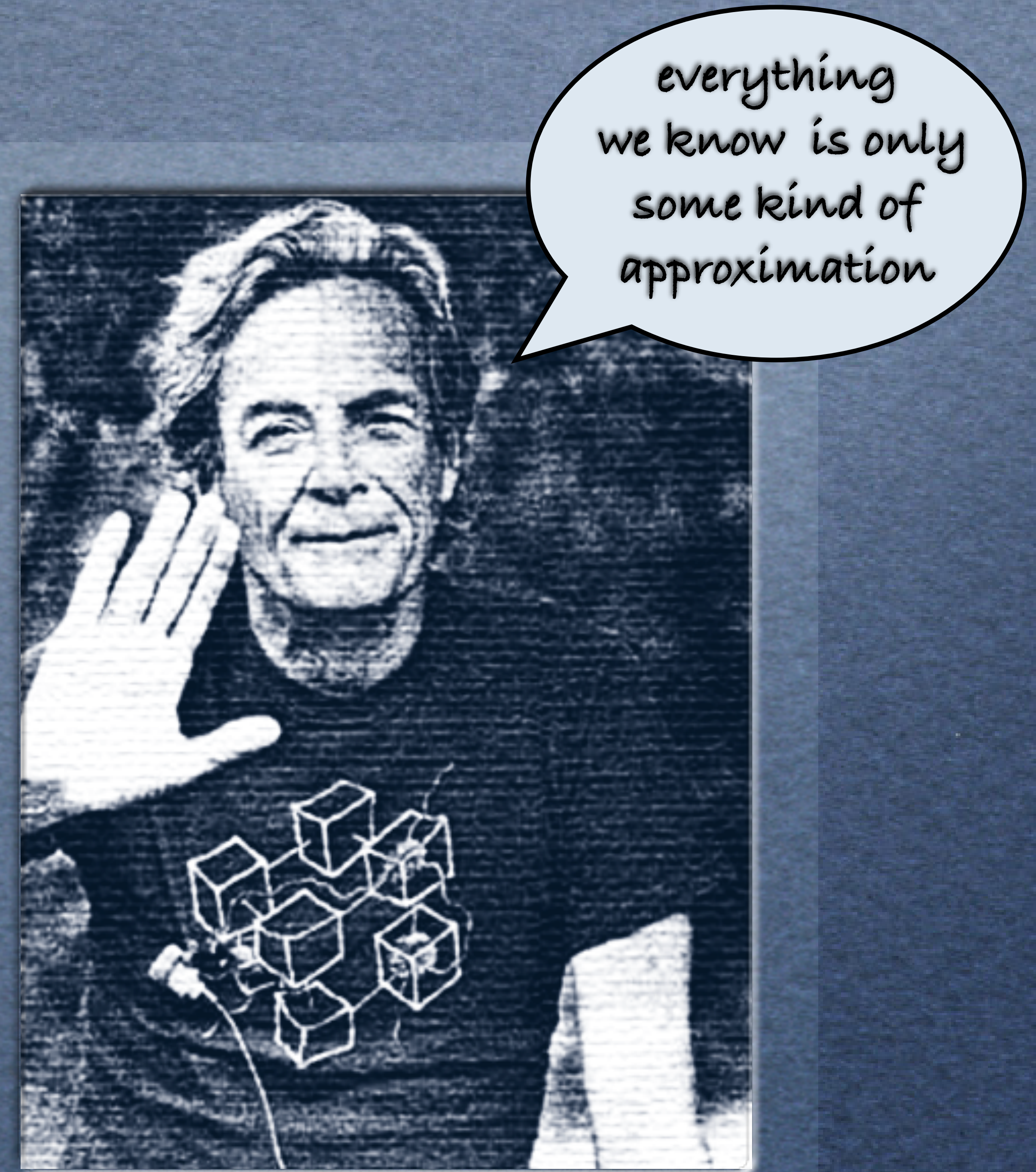
Plan of the talk

- What is Quantum Cosmology?
  Approximations in cosmology

- Definition of the theory

  1. *Kinematics*
     Graph expansion
  2. *Dynamics*
     Vertex expansion
  3. *Classical limit*
     Large volume expansion

- Summary and comments



The cosmological principle

- The dynamics of a homogeneous and isotropic space approximates well the observed universe.
- The effect of the inhomogeneities on the dynamics of its largest scale, described by the scale factor, can be neglected in a first approximation.
- This is not a large scale approximation, because it is supposed to remain valid when the universe was small! It is an expansion in $N \sim a/\lambda$!
- The full theory may be recovered by adding degrees of freedom one by one, starting from the cosmological ones.
- We can define an approximated dynamics of the universe for a finite number of degrees of freedom .

The cosmological principle

- The dynamics of a homogeneous and isotropic space approximates well the observed universe.
- The effect of the inhomogeneities on the dynamics of its largest scale, described by the scale factor, can be neglected in a first approximation.
- This is not a large scale approximation, because it is supposed to remain valid when the universe was small! It is an expansion in $N \sim a/\lambda$!
- The full theory may be recovered by adding degrees of freedom one by one, starting from the cosmological ones.
- We can define an approximated dynamics of the universe for a finite number of degrees of freedom .

Kinematics

Hilbert space: $\tilde{\mathcal{H}} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma$ where $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$

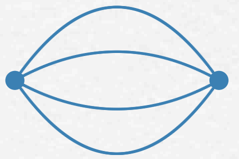
Identifications: $\tilde{\mathcal{H}}/\sim$

1. if Γ is a subgraph of Γ' then we must identify $\mathcal{H}_\Gamma$ with a subspace of $\mathcal{H}'_\Gamma$
2. divide $\mathcal{H}_\Gamma$ by the action of the discrete group of the automorphisms of Γ

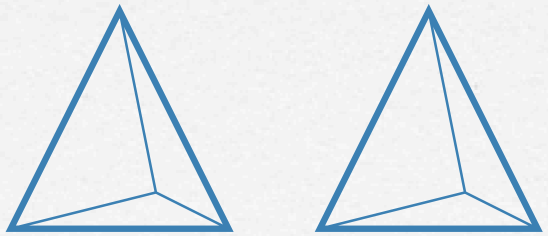
States that solve gauge constraint: $|\Gamma, j_\ell, v_n\rangle \in \tilde{\mathcal{H}} = \bigoplus_{\Gamma} \bigoplus_{j_\ell} \bigotimes_{n} \mathcal{H}_n$

Example:

the "dipole" where $N = 2$

$\Delta_2^* =$ 

so that Δ_2 is formed by two tetrahedra glued along all their faces = triangulated 3-sphere !

$\Delta_2 =$ 

Coherent states → Semiclassical States

$\mathcal{H}_\Gamma$ contains an (over-complete) basis of "wave packets" $\psi_{H_\ell} = \psi_{\vec{n}_\ell, \vec{n}'_\ell, \xi_\ell, \eta_\ell}$

Holomorphic-states:

$$\psi_{H_\ell}(U_\ell) = \int_{SU(2)^N} dg_n \bigotimes_{l\in\Gamma} K_t(g_{s(\ell)} U_\ell g_{t(\ell)}^{-1} H_\ell^{-1}).$$

superpositions of SN states

"group averages" on the gauge invariant states

$$H = D^{\frac{1}{2}}(R_{\vec{n}})\, e^{-i(\xi+i\eta)\frac{\sigma_3}{2}}\, D^{\frac{1}{2}}(R_{\vec{n}'}^{-1})$$

$H_\ell \in SL(2,\mathbb{C})$

heat kernel peaks each U_ℓ on H_ℓ

Geometrical interpretation

for the $(\vec{n}, \vec{n}', \xi, \eta)$ labels:

$\vec{n}, \vec{n}'$ are the 3d normals to the faces of the cellular decomposition

$\xi \leftrightarrow$ extrinsic curvature at the faces and

$\eta \leftrightarrow$ area of the face divided by $8\pi G$.

Choose coherent states $|H_\ell\rangle$ describing a homogeneous and isotropic geometry:

$$z_\ell = \xi_\ell + i\eta_\ell$$
$$\downarrow$$
$$z = \alpha c + i\beta p \qquad \forall \ell$$

Graph Expansion

Mode expansion $\longleftrightarrow$ truncation on a graph

- **Restrict the states to a fixed graph with a finite number N of nodes.**
  This defines an approximated kinematics of the universe, inhomogeneous but truncated at a finite number of cells.

- **The graph captures the large scale d.o.f. obtained averaging the metric**
  over the faces of a cellular decomposition formed by N cells.

- **The full theory can be regarded as an expansion for growing N.**
  FRW cosmology corresponds to the lower order where there is only a regular cellular decomposition: the only d.o.f. is given by the volume.

- **Coherent states $|H_\ell\rangle$ describing a homogeneous and isotropic geometry:**

$$z = \xi_\ell + i\eta_\ell \quad \longrightarrow \quad z = \alpha c + i\beta p \qquad \forall \ell$$

  Geometry is determined by (c, p) in the past and (c', p') in the future.

Dynamics

see Rovelli's talk

The spinfoam formalism associates an amplitude to each boundary state $\psi \in \mathcal{H}$.

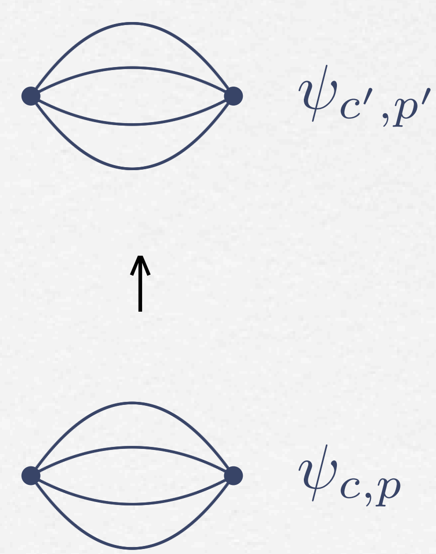
$$\langle W \mid \psi \rangle = \sum_{\sigma} \prod_{f} d_f(\sigma) \prod_{v} W_v(\sigma)$$

$$W_v(H_\ell) = \int_{SO(4)^N} dG_n \;\; \prod_{\ell} P_t(H_\ell \,,\, G_{s(\ell)} G^{-1}_{t(\ell)})$$

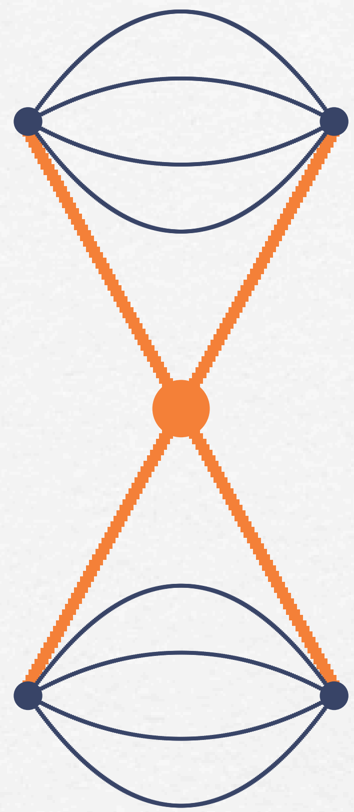
where

$$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)} \mathrm{Tr}\left[D^{(j)}(H) Y^\dagger D^{(j^+,j^-)}(G) Y \right]$$

In cosmology:
transition 3-sphere → 3-sphere



Vertex expansion



$\langle W|\psi_{H_{l(z,z')}}\rangle = W(z,z')$

At 1st order in the vertex expansion,
the "dipole-dipole" amplitude
is given by the spinfoam :

Large-distance expansion:

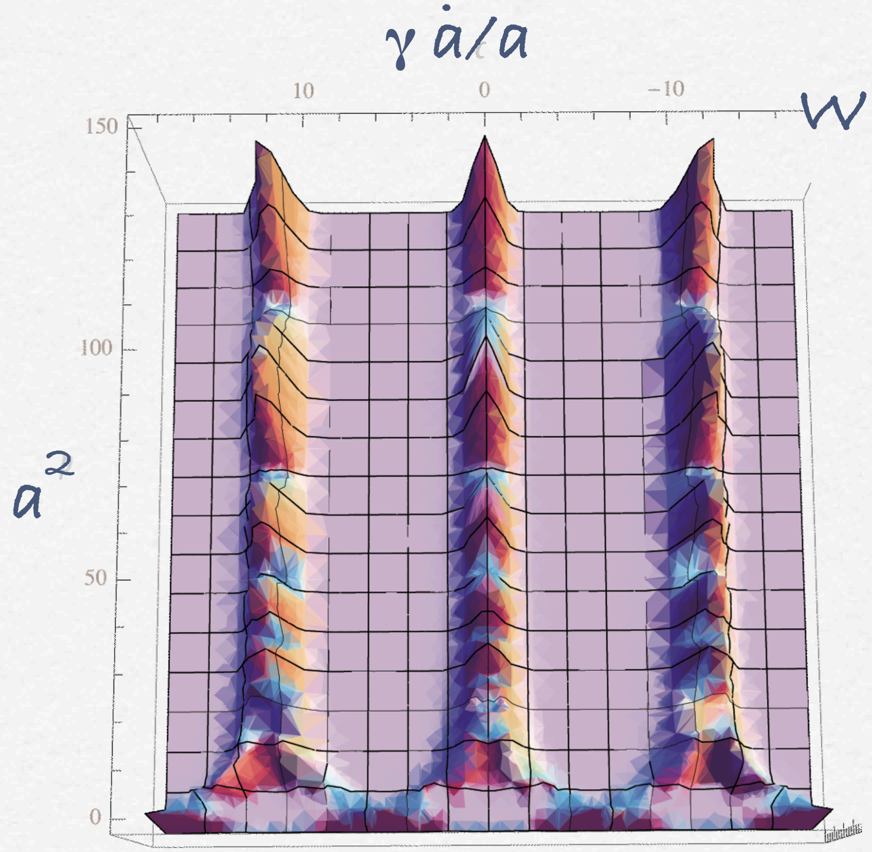
Boundary state peaked on boundary geometry large compared with the Planck length.
Holomorphic boundary states ψ_{H_ℓ} where $\eta_\ell >> 1$ in each H_ℓ .

This can be computed explicitly! Bianchi, Rovelli, FV

$$W(z,z') = C\ zz' e^{-\frac{z^2+(z')^2}{\hbar}}$$

Classical limit 1

At fixed (c′,p′), the transition amplitude W is peaked on the following values of (c,p)



Work in progress !!!

The peaks correspond to the (semi) classical trajectories

Classical limit 2

$z = \alpha c + i\beta p$

This resulting amplitude happens to satisfy an equation

$$\hat{H}(z, \frac{d}{dz})W(z', z) = \frac{3}{8\pi G(4\alpha\beta\gamma)^2}\left(\hat{z}^2 - \hat{\bar{z}}^2 - 2\right)^2 W(z', z) = 0.$$

In terms of (c,p) variables:

$$H = \frac{3}{8\pi G(4\alpha\beta\gamma)^2}\left(4i\,\alpha\beta\,cp - 2\hbar\right)^2 = 0$$

In the large p limit and dividing by $Vol \sim p^{3/2} > 0$ we have:

$$H = -\frac{3}{8\pi G\gamma^2}\,\sqrt{p}c^2 = 0$$

This is precisely the hamiltonian constraint of a homogeneous and isotropic cosmology.

→ LQG yields the Friedmann equation.

Classical limit 2

$z = \alpha c + i\beta p$

This resulting amplitude happens to satisfy an equation

$$\hat{H}(z, \frac{d}{dz})W(z', z) = \frac{3}{8\pi G(4\alpha\beta\gamma)^2}\left(\hat{z}^2 - \hat{\bar{z}}^2 - 2\right)^2 W(z', z) = 0.$$

In terms of (c,p) variables:

$$H = \frac{3}{8\pi G(4\alpha\beta\gamma)^2}\left(4i\,\alpha\beta\,cp - 2\hbar\right)^2 = 0$$

In the large p limit and dividing by $Vol \sim p^{3/2} > 0$ we have:

$$H = -\frac{3}{8\pi G}\; a\dot{a}^2 = 0$$

This is precisely the hamiltonian constraint of a homogeneous and isotropic cosmology.

→ LQG yields the Friedmann equation.

Summary

1. It is possible to compute quantum transition amplitudes explicitly

   in suitable approximations:

   1 graph expansion
   2 vertex expansion
   3 large volume expansion

2. The transition amplitude computed appears to give the correct

   Friedmann dynamics in the classical limit.

3. Family of models opening a systematic way

   - for describing the inhomogeneous d.o.f. in quantum cosmology,
   - for studying the fluctuations of quantum geometry at the bounce.

4. Light on LQC/LQG relation.

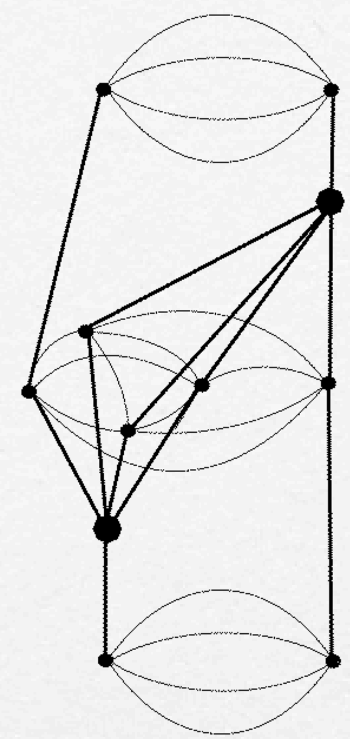
Work in progress

- Lorentzian version
  Cosmological Constant
  Matter

work in progress !!!

Bianchi, Magliaro, Marcianò, Perini, Rovelli, FV...

- Further order in the vertex expansion → → →



- Many nodes; Many edges ($U(N)$ simmetry) Borja, Diaz-Polo, Garay, Livine.

- Relation to Loop Quantum Cosmology

also to Ashtekar, Campiglia, Henderson SF expansion.

- Compute quantum corrections: does bounce scenario survives?

- How do quantum inhomogeneous fluctuations affect structure formation? and inflation?

... and more!