What's the entropy of gravity?

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things you know	Loop Quantum Gravity
Yang-Mills Theory electric field, vector potential	Gravity as a $SU(2)$ gauge theory $(E^{ai}(x), A^i_a(x))$ $q^{ab}(X) = \frac{E^{ai}(X)E^{bi}(X)}{q(X)}$
angular momentum $L^2 \lvert j, m\rangle = \hbar^2 j(j+1) \lvert j, m\rangle$	area $E_{i\ell}E^i_\ell \mid s\,\rangle = (8\pi\gamma\, G\hbar)^2\, j_\ell(j_\ell + 1) \mid s\,\rangle$ $\sqrt{q(x)} \mid s\,\rangle = \sum_{n \in N(s)} \nu_n\, \delta(x, x_n) \mid s\,\rangle$
states	$\mid s\,\rangle := \mid \Gamma, j_\ell, \nu_n\,\rangle$
Hilbert space	$\mathcal{H}_{\text{LQG}} = \bigoplus_\Gamma\ \mathcal{H}_\Gamma$

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References:

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$\Gamma = (N, L)$ undirected simple graph

Adjacency $[A(\Gamma)]_{n,m} = 1$ if $\{n, m\} \in L(\Gamma)$ and $[A(\Gamma)]_{u,v} = 0$ otherwise

Degree $[\Delta(\Gamma)]_{n,n} := d_n$ $= \#$ links adjacent to the node n

Laplacian $L(\Gamma) := \Delta(\Gamma) - A(\Gamma)$

Density $\rho_\Gamma := \frac{L(\Gamma)}{d_\Gamma} = \frac{L(\Gamma)}{\mathrm{Tr}(\Delta(\Gamma))}$ Hermitian, positive semi-definite, trace-1

Entropy $S(\Gamma) = -\mathrm{Tr}[\rho_\Gamma \log \rho_\Gamma]$

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Plan of the talk

1. Interaction Hamiltonian
   single quantum particle on the quantum gravitational field
   - construction and proprieties -

2. comparison with Graph Theory
   our Hamiltonian provides (up to a certain approximation)
   the same objects already studied in Graph Theory

3. Loop Quantum Thermodynamics
   this Hamiltonian is a tool to construct statistical objects in LQG

connection variables

Phase Space $(q_{ab}(x), \pi^{ab}(x), X^a, P_a)$

Hamiltonian constraint $C(x) = H_{ADM}(x) + \delta^3(x, X) P_0$ $P_0 = \sqrt{P^2 + m^2} \sim m + \frac{P^2}{2m}$

$P^2 = q^{ab}(x) P_a P_b$

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$$H = \int \mathrm{d}x\ f_R(x,X)\frac{E^{ai}(x)E^{bi}(X)}{\sqrt{q(X)}}\frac{P_a\,P_b}{2m}$$

pointlike nature of the particle $\rightarrow$ regularization by a smearing function

$$f_R(x,X) = \begin{cases} \frac{1}{V_R} = \frac{3}{4\pi R^3} & \text{if} \quad |x-X| \leq R \\ 0 & \text{if} \quad |x-X| \geq R \end{cases}$$

Quantum States

Spin network states $|\, s, x \,\rangle \equiv |\, s \,\rangle \otimes |\, x \,\rangle \subset \mathcal{H}_{\mathrm{LQG}} \otimes \mathcal{H}_{\mathrm{P}}$.

$$\mathbb{I}_{\mathrm{P}} = \int \mathrm{d}x \quad |\, x \,\rangle\langle\, x \,|$$

$$\langle\, x \,|\, y \,\rangle = \quad \delta(x, y)$$

the volume operator vanishes everywhere except at the nodes

$$\sqrt{q(x)}\,|\, s \,\rangle = \sum_{n \in N(s)} \nu_n \, \delta(x, x_n)\,|\, s \,\rangle$$

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restriction to the nodes

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$$\langle\, x\,|\, y\,\rangle = \frac{1}{\sqrt{q(x)}}\;\; \delta(x, y)$$

$$\langle\, s, x_n\,|\, s', x_{n'}\,\rangle = \nu_n^{-1}\, \delta_{ss'}\,\delta_{nn'} \qquad \mathbb{I} = \textstyle\sum_s \sum_{n \in N(s)} \nu_n\, |\, s, x_n\,\rangle\langle\, s, x_n\,|$$

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Quantum States

for later convenience... $|\underset{\sim}{x}\rangle := \sqrt[4]{q(x)}\,|x\rangle$

Spin network states $\quad |s,x\rangle \equiv |s\rangle \otimes |x\rangle \subset \mathcal{H}_{\rm LQG} \otimes \mathcal{H}_{\rm P}$.

$$\mathbb{I}_{\rm P} = \int \mathrm{d}x \, \sqrt{q} \;\; |x\rangle\langle x| \;\; \rightarrow \;\; \langle s|\mathbb{I}_{\rm P}|s\rangle = \sum_{n \in \underset{\sim}{N}(s)} \nu_n^2 \, |\underset{\sim}{x}_n\rangle\langle \underset{\sim}{x}_n|$$

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Quantum Operators

$$E^{ai}(x)\,|\,s\,\rangle \;=\kappa\hbar\sum_\ell \int_\ell \mathrm{d}t\,\dot{\ell}^a(t)\,\delta^3(x,\ell(t))\,|\,s,\tau^i\,\rangle$$

$$E^{ai}(x)E^{bi}(x)\,|\,s\,\rangle \;= \\ (\kappa\hbar)^2\sum_{\ell,\ell'}\;\int_\ell \mathrm{d}t\;\int_{\ell'}\mathrm{d}t'\;\;\dot{\ell}^a(t)\,\dot{\ell}^b(t')\;\delta^3(x,\ell(t))\delta^3(x,\ell(t'))j_\ell(j_\ell+1)\,|\,s\,\rangle$$

$$P_a = -i\hbar\; D_a$$ covariant derivative

$$\langle\, s,\psi\,|H|\, s,\phi\,\rangle = \frac{\kappa^2\hbar^4}{2m} \sum_{\ell} j_\ell(j_\ell+1)\int_\ell \mathrm{d}s \int_\ell \mathrm{d}t\ \overline{\partial_s\psi(\underset{\sim}{\ell(s)})}\,\partial_t\phi(\underset{\sim}{\ell(t)})\ f_R(\ell(s),\ell(t))$$

Planck scale! $\Delta_\ell\psi := \int_\ell \mathrm{d}s\ \partial_s\psi(\underset{\sim}{\ell(s)}) = \psi(\underset{\sim}{\ell_f}) - \psi(\underset{\sim}{\ell_i})$

particle states

$$|\,\underset{\sim}{\ell}\,\rangle := |\,\underset{\sim}{\ell_f}\,\rangle - |\,\underset{\sim}{\ell_i}\,\rangle$$

$$H = \frac{(8\pi\hbar)^2\ell_{\mathrm{Pl}}^4}{2m\, v_R} \sum_{s,\ell\in s} j_\ell(j_\ell+1)\ |\, s,\underset{\sim}{\ell}\,\rangle\langle\, s,\underset{\sim}{\ell}\,|$$

trace-1 $\mathrm{Tr}H = \frac{\hbar^2\ell_{\mathrm{Pl}}^4}{2m^*} \sum_{n\in N} v_n \sum_{\ell\in n} j_\ell(j_\ell+1)$

$$\longrightarrow \quad \rho = \frac{H}{\mathrm{Tr}H} = \frac{\sum_{\ell\in n} j_\ell(j_\ell+1)}{\sum_{n\in N} v_n \sum_{\ell\in n} j_\ell(j_\ell+1)}\,|\,\underset{\sim}{\ell}\,\rangle\langle\,\underset{\sim}{\ell}\,|$$

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Approximation! j_ℓ all the same v_n all the same

Adjacency

$$\begin{aligned}\langle\, n\,|\rho|\; m\,\rangle_{ph} &= \frac{1}{d_\Gamma}\sum_\ell (\delta_{n,\ell_f}-\delta_{n,\ell_i})(\delta_{m,\ell_f}-\delta_{m,\ell_i}) \\ &= \frac{1}{d_\Gamma}(-1) \text{ if } \{n,m\}\in S \text{ and } [A(\Gamma)]_{u,v}=0 \text{ otherwise}\end{aligned}$$

Degree

$$\langle\, n\,|\rho|\; n\,\rangle_{ph} = \frac{1}{d_\Gamma}\sum_\ell (\delta_{n,\ell_f}-\delta_{n,\ell_i})^2 = \frac{d_n}{d_\Gamma}$$

Entropy

$$S = -\mathrm{Tr}[\rho\log\rho] \equiv S_{BGS}$$

Approximation! j_ℓ all the same v_n all the same

Adjacency

$$\begin{aligned} \langle\, n \,|\rho|\, m \,\rangle_{ph} &= \frac{1}{d_\Gamma} \sum_\ell (\delta_{n,\ell_f} - \delta_{n,\ell_i})(\delta_{m,\ell_f} - \delta_{m,\ell_i}) \\ &= \frac{1}{d_\Gamma}(-1) \text{ if } \{n, m\} \in S \text{ and } [A(\Gamma)]_{u,v} = 0 \text{ otherwise} \end{aligned}$$

Degree

$$\langle\, n \,|\rho|\, n \,\rangle_{ph} = \frac{1}{d_\Gamma} \sum_\ell (\delta_{n,\ell_f} - \delta_{n,\ell_i})^2 = \frac{d_n}{d_\Gamma}$$

Entropy

$$S = -\mathrm{Tr}[\rho \log \rho] \equiv S_{BGS}$$

$< E > = \mathrm{Tr}_{\mathrm{LQG}}[H' \tilde{\rho}] = -d(\ln Z)/d\mu$ mean energy of the particle on a gravitational field

from a measure of the particle (geometry known) | from a measure of the geometry (particle position known)

Jayne's principle of maximum entropy $S = -\mathrm{Tr}_{\mathrm{LQG}}[\tilde{\rho} \log \tilde{\rho}] = \log Z + \mu\, \mathrm{Tr}_{\mathrm{LQG}}[H' \, e^{-\mu H'}]$

Partition function: $Z = \mathrm{Tr}_{\mathrm{LQG}}[e^{-\mu H'}]$

$Z = \sum_s e^{-\frac{E_s}{kT}}$ where $E_s = E_0 d_s$ and take $\frac{E_0}{kT} := \mu$ $\rightarrow$ $Z(\mu) = \sum_s e^{-\mu d_s}$

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$|\psi\rangle \in \mathcal{H}_{\mathrm{P}}$ $\quad H' = \langle \psi | H | \psi \rangle$ operator in $\mathcal{H}_{\mathrm{LQG}}$ $\quad \tilde{\rho} = \frac{1}{Z(\mu)} \, e^{-\mu H'}$

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Partition Function

$$Z(\mu) \;=\; \sum_s e^{-\mu \bar{d}_s} \qquad \bar{d}_s(\mu) = \frac{\sum_n^N d_n}{N} = \frac{2\ell}{N}$$

Energy Density

$$\rho_s(\mu) \;=\; \frac{1}{Z(\mu)} e^{-\mu \bar{d}_s} \;=\; e^{-\frac{2}{N}\ell}\left(1 + e^{-\mu\frac{2}{N}}\right)^{-L}$$

$$< d > = \frac{1}{Z(\mu)} \sum_s \bar{d}_s e^{-\mu \bar{d}_s} \;=\; -\frac{1}{Z(\mu)} \frac{d}{d\mu} Z(\mu) \;=\; \frac{2}{N} L \left(1 + e^{+\mu\frac{2}{N}}\right)^{-1}$$

$$\Delta d \;= < d^2 > - < d >^2 = -\frac{4L}{N^2} e^{+\mu\frac{2}{N}} \left(1 + e^{+\mu\frac{2}{N}}\right)^{-2}$$

Entropy

$$S \;=\; \mu < d > - \ln Z(\mu) \;=\; \mu \frac{2}{N} L \left(1 + e^{+\mu\frac{2}{N}}\right)^{-1} - L \ln\left(1 + e^{-\mu\frac{2}{N}}\right)$$

Partition Function $$Z(\mu) = \sum_s e^{-\mu \bar{d}_s} = \left(1 + e^{-\mu \frac{2}{N}}\right)^L$$

Energy Density $$\rho_s(\mu) = \frac{1}{Z(\mu)} e^{-\mu \bar{d}_s} = e^{-\frac{2}{N}\ell} \left(1 + e^{-\mu \frac{2}{N}}\right)^{-L}$$

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Entropy

$$S = \mu < d > - \ln Z(\mu) = \mu \frac{2}{N} L \left(1 + e^{+\mu \frac{2}{N}}\right)^{-1} - L \ln \left(1 + e^{-\mu \frac{2}{N}}\right)$$

Summary

- Hamiltonian of a non-relativistic particle on a gravitational field described by a spinnetwork;
- we have calculated a partition function, and from this other statistical quantities... is this a first step through a viable thermodynamics of the gravitational field?

"Single particle in quantum gravity and BGS entropy of a spin network" by C. Rovelli and FV. Phys.Rev.D81:044038,2010 (arXiv:0905.2983)