Spinfoam and Cosmology

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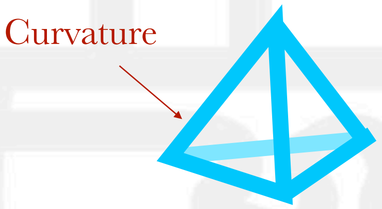
HISTORY OF THE MAIN IDEAS

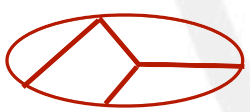
old quantum gravity

- 1957 $Z(q) = \int_{\partial g = q} Dg \;\; e^{iS_{EH}[g]}$ [Misner]
- 1961 Regge calculus → truncation of GR [Regge]
- 1967 W-DeW equation [Wheeler, DeWitt]
- 1971 Spin-geometry theorem → spin network [Penrose]



old LQG

- 1988 Complex variables for GR [Ashtekar]
- 1988 Loop solutions to WdW eq → LQG [Rovelli-Smolin]
- 1994 Spectral problem for geometrical operators → spin network



- 1996 Covariant dynamics → spinfoams [Reisenberger-Rovelli]

- 1999 LQC [Bojowald]

new results

- 2008 Covariant dynamics of LQG [Engle-Pereira-Livine-Rovelli, Freidel-Krasnov]
- 2010 Asymptotic of the new dynamics → recovery of Regge action [Conrady-Freidel, Barrett et al, Bianchi]
- 2011 Cosmological constant → finiteness of the transition amplitudes [Han, Fairbairn-Moesburger]

→

- 2010 SPINFOAM COSMOLOGY [Bianchi-Rovelli-FV]

- (canonical) LQC

Input:

- SU(2) group variables
  - Minimal area gap
- Hamiltonian constraint
  - Holonomy corrections
  - Inverse-volume corrections

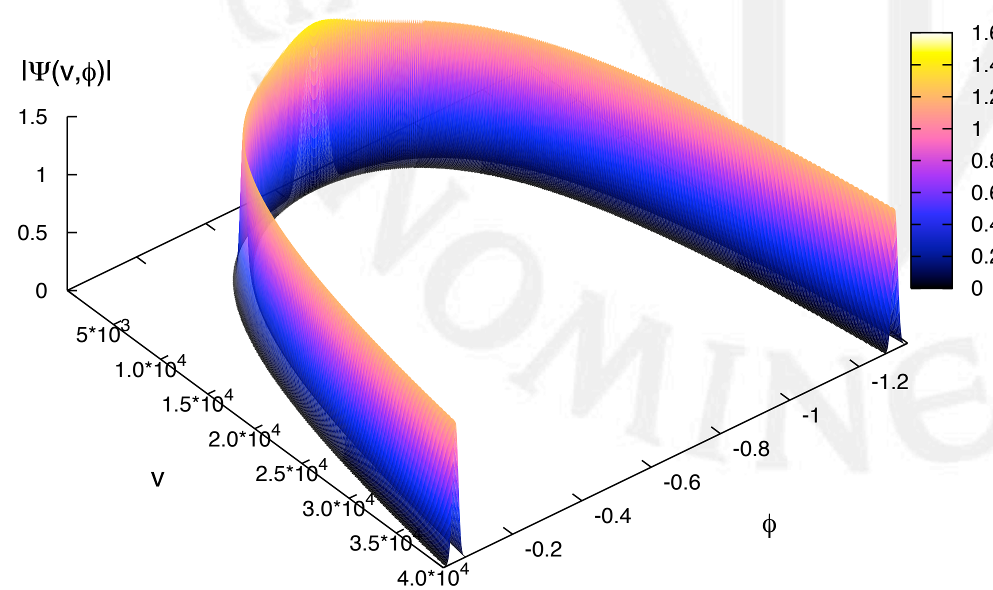
Output:

- Singularity resolution
  - No need to violate the SEC
- Modified Friedmann equations
  - Wave-packet non-singular trajectories
- Modified Muhanov-Sasaki equations
  - Predictions for the CMB

- $$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho\left(1-\frac{\rho}{\rho_c}\right)$$
- $$v'' - \left(1 - 2\frac{\rho}{\rho_c}\right)\nabla^2 v - \frac{z''}{z}v = 0$$

- Barrau, Cailleteau, Grain, Mielczarek, Linsefors
- Agullo, Ashtekar, Nelson

- Ashtekar, Pawlowski, Singh



IN THIS TALK

- (covariant) LQG

Input:

- Dynamics of quanta of spacetime
  - Group variables on graphs
- Lorentzian signature
- Local product of interaction vertex
  - Feynman rules

Output:

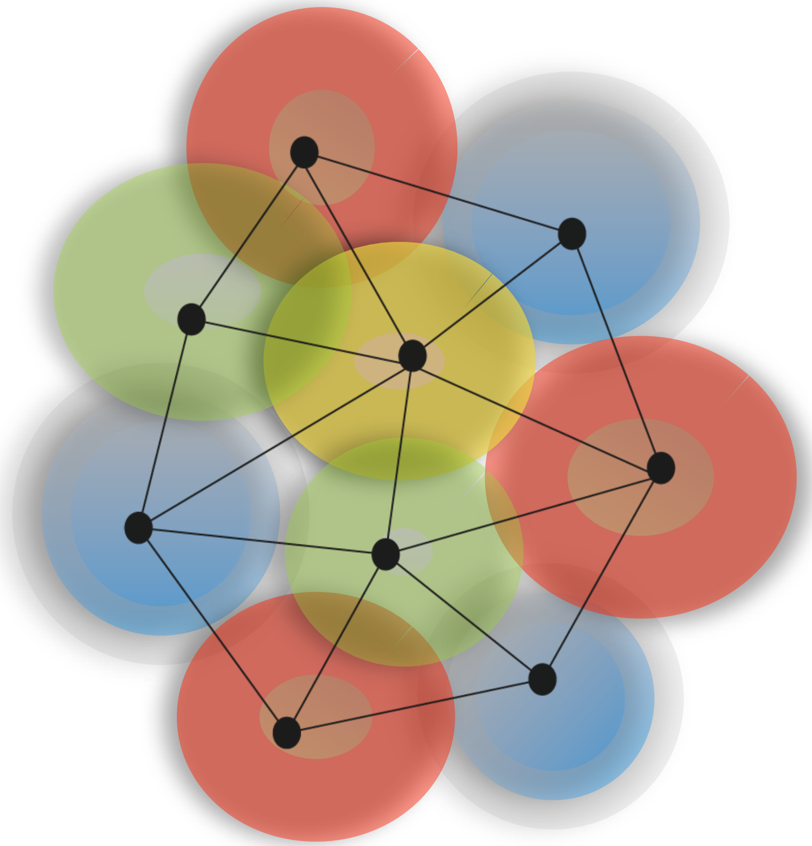
- UV finite
  - Physical cutoff at the Planck scale
- IR finite
  - Cosmological const. = q-deformation
- GR recover in the semiclassical limit!
- Easy to couple YM fields

- COSMOLOGY

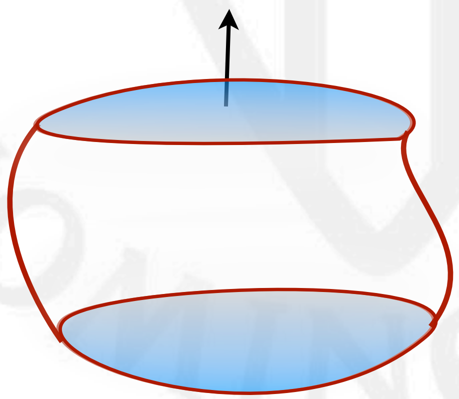
- Cosmology provides the (only?) ground to check the theory
- A quantum cosmology based on the full quantum theory
  - Possibility to explore the deep quantum regime in the early universe
  - Possibility to include quantum fluctuations naturally

prejudice 1

“in quantum gravity spacetime should be quantized”

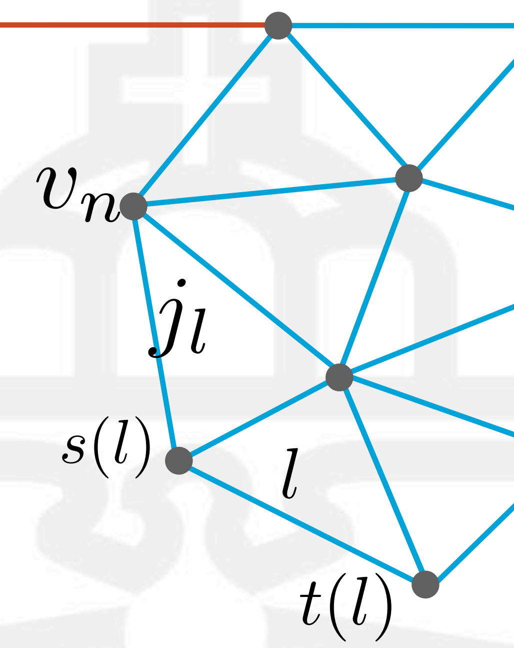


- It is a theory about quanta of spacetime
- Each quantum is Lorentz invariant
- The states are boundary states at fixed time
- The physical phase space is spanned by SU(2) group variables

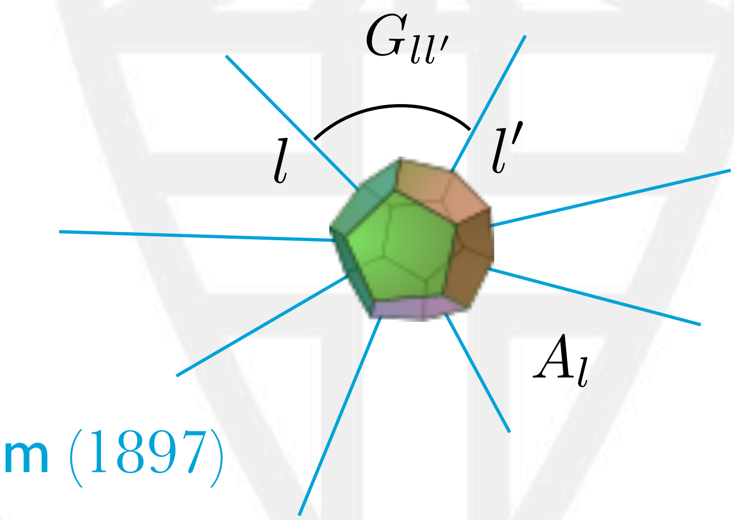


$SL(2, \mathbb{C}) \to SU(2)$

HILBERT SPACE & OPERATOR ALGEBRA



- Abstract graphs: $\Gamma=\{N,L\}$
- Group variables: $\begin{cases} h_l \in SU(2) \\ \vec{L}_l \in su(2) \end{cases}$
- Graph Hilbert space: $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$
- The space $\mathcal{H}_\Gamma$ admits a basis $|\Gamma, j_\ell, v_n\rangle$
- Gauge invariant operator $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ with $\sum_{l\in n} G_{ll'} = 0$



Penrose's **spin-geometry theorem** (1971), and **Minkowski theorem** (1897)

- h_l "Holonomy of the Ashtekar-Barbero connection along the link"
- $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ SU(2) generators $\quad L^i\psi(h) \equiv \left.\frac{d}{dt}\psi(he^{t\tau_i})\right|_{t=0}$

  gravitational field operator (tetrad)

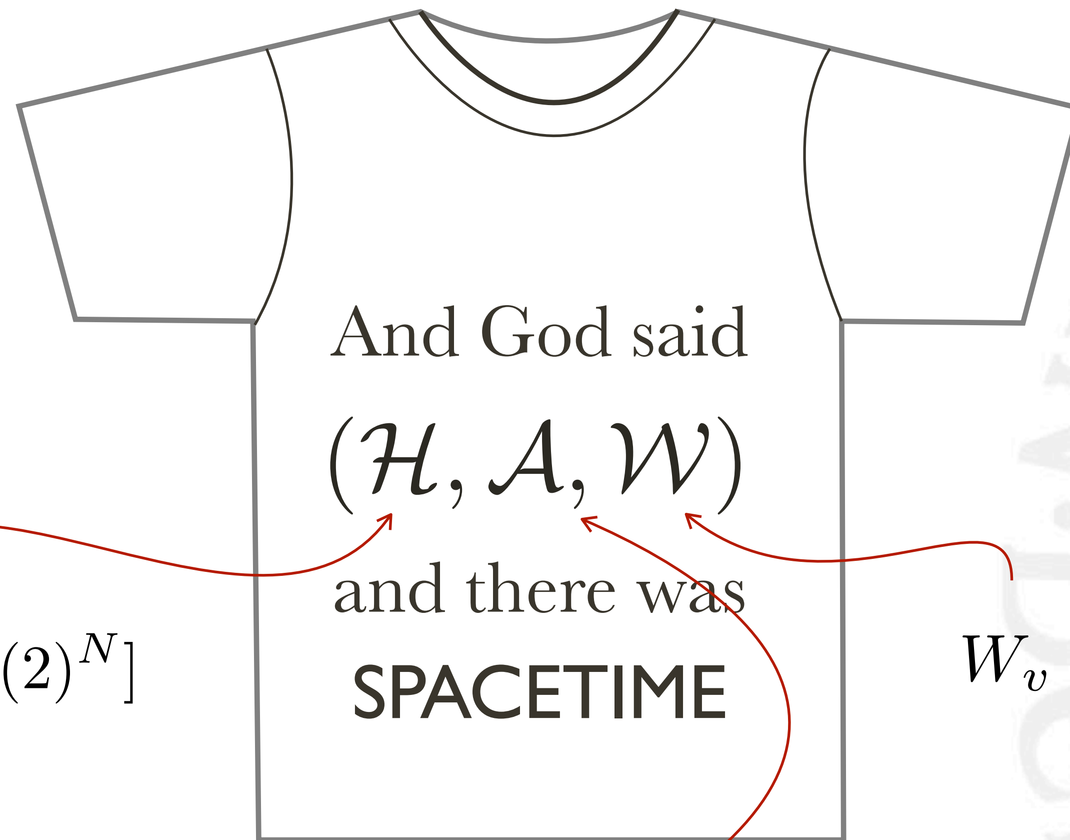
- Composite operators:
  - Area: $$A_\Sigma = \sum_{l\in\Sigma} \sqrt{L_l^i L_l^i}.$$
  - Volume: $$V_R = \sum_{n\in R} V_n, \qquad V_n^2 = \frac{2}{9}\,|\epsilon_{ijk} L_l^i L_{l'}^j L_{l"}^k|.$$
  - Angle: $$L_l^i L_{l'}^i$$

- Geometry is quantized:
  - eigenvalues are discrete
  - the operators do not commute
  - quantum superposition
    → *coherent states*

Quantum states of space, rather than states on space.

prejudice 2

“the theory should be so simple and short that it would fit on a tshirt”



Hilbert Space:
$\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$

Transition Amplitude:
$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\ \psi_v)(\mathbb{1})$

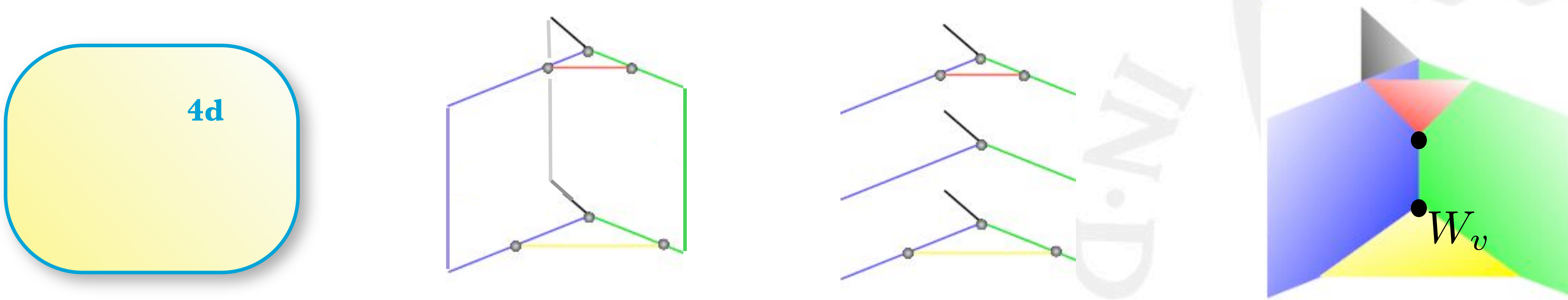
Operator Algebra:
$[L_a^i, L_b^j] = i\delta_{ab}\ell^2\epsilon^{ij}_k L_a^k$

$(\mathcal{H}, \mathcal{A}, \mathcal{W})$ defines a background independent quantum field theory

[Engle-Pereira-Livine-Rovelli, Freidel-Krasnov '08]

Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

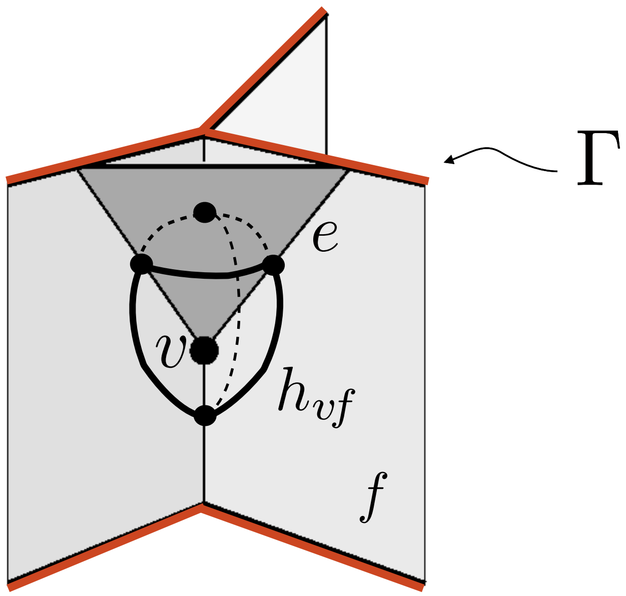
Amplitude associated to a state ψ of a boundary of a 4d region



3d boundary — boundary graph — a spin network history — σ : spinfoam

- Superposition principle $\langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Locality: vertex amplitude $W(\sigma) \sim \prod_v W_v.$
- Lorentz covariance $W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\ \psi_v)(\mathbb{1})$

2-complex $\mathcal{C}$
(vertices, edges, faces)



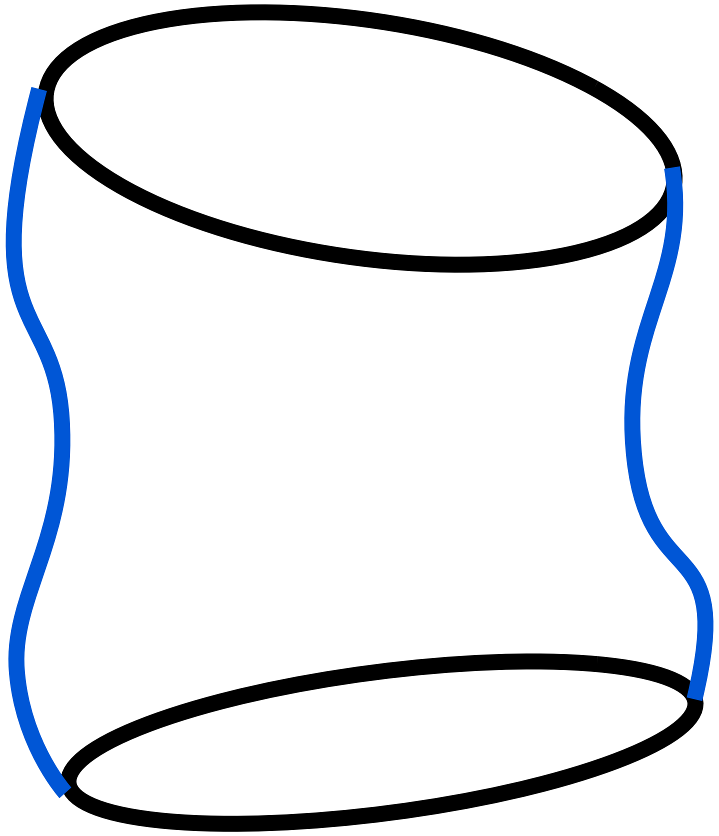
- Transition amplitudes $W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$ $\qquad h_f = \prod_v h_{vf}$

- Vertex amplitude $A(h_f) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f (2j_f+1)\ Tr_j[h_f Y_\gamma^\dagger g_e g_{e'}^{-1} Y_\gamma]$

- Simplicity map
$$Y_\gamma \ : \ \mathcal{H}_j \quad \to \quad \mathcal{H}_{j,\gamma j}$$
$$|j;m\rangle \ \mapsto \ |j,\gamma(j+1);j,m\rangle$$

COSMOLOGIACAL TRANSITION AMPLITUDES

[Rovelli, FV]



q'_{ij}

the universe at “time” t’

q_{ij}

the universe at “time” t

$$W(q'_{ij}, q_{ij}) \sim \int_{\partial g = q', q} Dq \; e^{i\,S}$$

- Fixed graph with N of nodes. This defines an approximated kinematics of the universe, inhomogeneous but truncated at a finite number of cells.
- The graph captures the large scale d.o.f. obtained averaging the metric over the faces of a cellular decomposition formed by N cells.

- The full theory can be regarded as an expansion for growing N. FRW cosmology corresponds to the lower order where there is only a regular cellular decomposition: the only d.o.f. is given by the volume.
- Coherent states $|H_\ell\rangle$ describing a homogeneous and isotropic geometry:

$$z = \xi_\ell + i\eta_\ell \quad \longrightarrow \quad z = \alpha \dot{R} + i\beta R^2 \qquad \forall \ell$$

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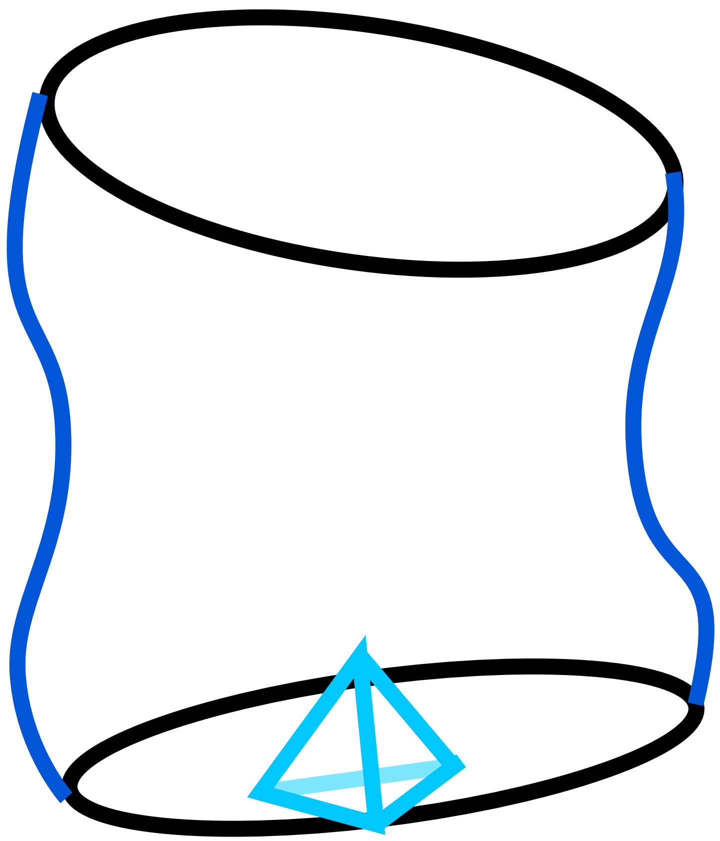

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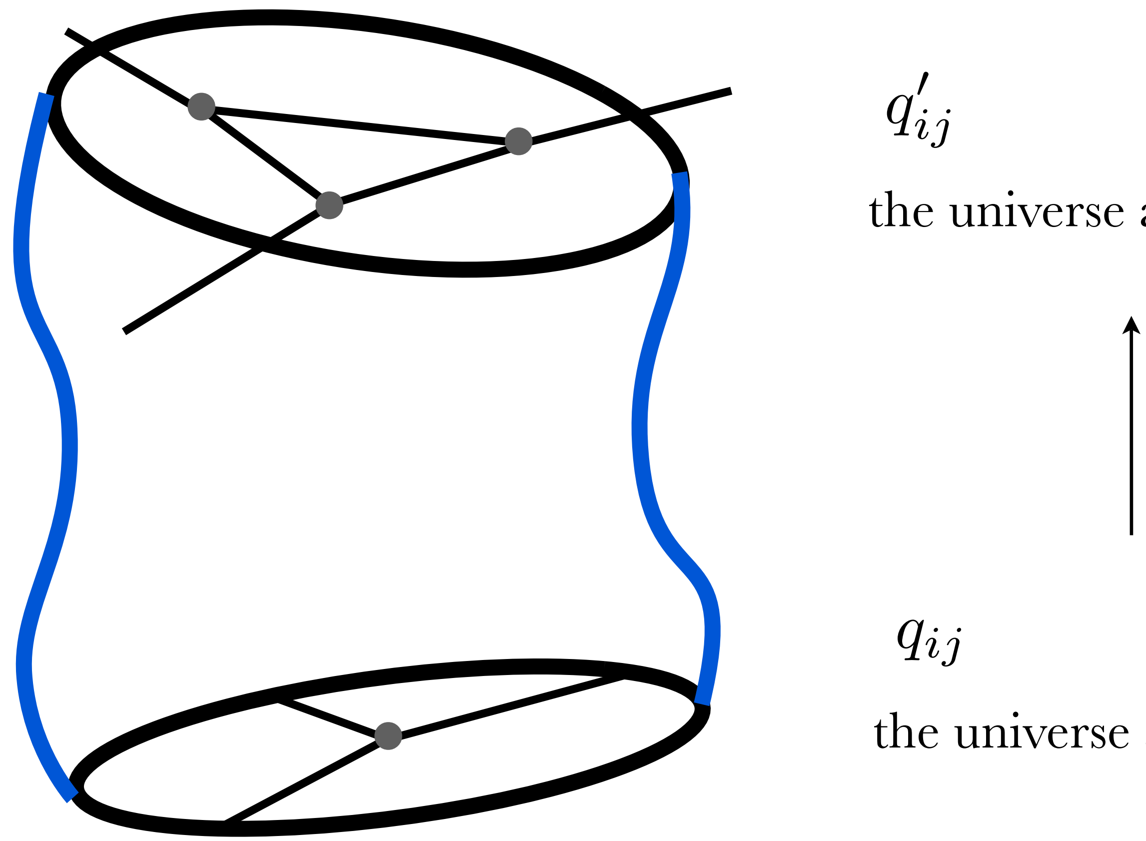

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Tetrads $g_{ab} \to e^i_a$ $g_{ab} = e^i_a \, e^i_b$ $e = e_a dx^a \in \mathbb{R}^{(1,3)}$

Spin connection $\omega = \omega_a dx^a \in sl(2,\mathbb{C})$ $\omega(e):$ $de + \omega \wedge e = 0$

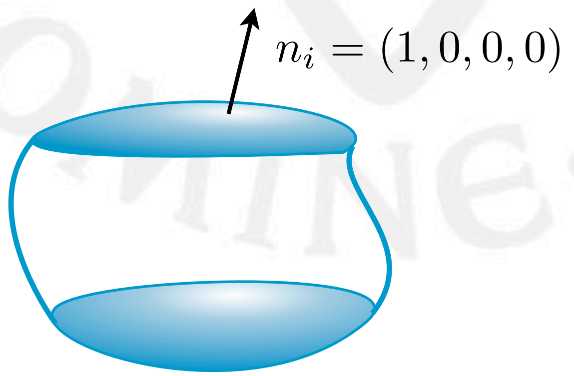
GR action $$S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma} \int e \wedge e \wedge F[\omega]$$ *(Holst term)*

BF theory $$S[e,\omega] = \int B[e] \wedge F[\omega]$$

Canonical variables $$\omega, \quad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$$

On the boundary $n_i = e^a_i n_a$ $n_i e^i = 0$ $SL(2,\mathbb{C}) \to SU(2)$

$$B \to (K = nB, L = nB^*)$$



Linear simplicity constraint $\vec{K} + \gamma \vec{L} = 0$

$SL(2,\mathbb{C})$ UNITARY IRRIDUCIBLE REPRESENTATIONS

$SU(2)$ unitary representations: $2j \in Z$ $\quad |j;m\rangle \in \mathcal{H}_j$

$SL(2,\mathbb{C})$ unitary representations: $2k \in N, \quad \nu \in R$ $\quad |k,\nu;j,m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,\nu}$

γ-simple representations: $\nu = \gamma(k+1)$

$SU(2) \rightarrow SL(2,\mathbb{C})$ map:

$$Y_\gamma : \quad \mathcal{H}_j \quad \rightarrow \mathcal{H}_{j,\gamma j}$$
$$|j;m\rangle \mapsto |(j,\gamma(j+1))\,;\; j,m\rangle$$

Image of Y_γ : $j = k$ *Langlands classification: Vogan's minimal k-type)*

Main property: $\vec{K} + \gamma\vec{L} = 0$ weakly on the image of Y_γ

Boost generator (→ $\vec{K}$) Rotation generator (→ $\vec{L}$)

prejudice 3

"quantum gravity
should remove the infinities
of general relativity"

[Rovelli, FV]

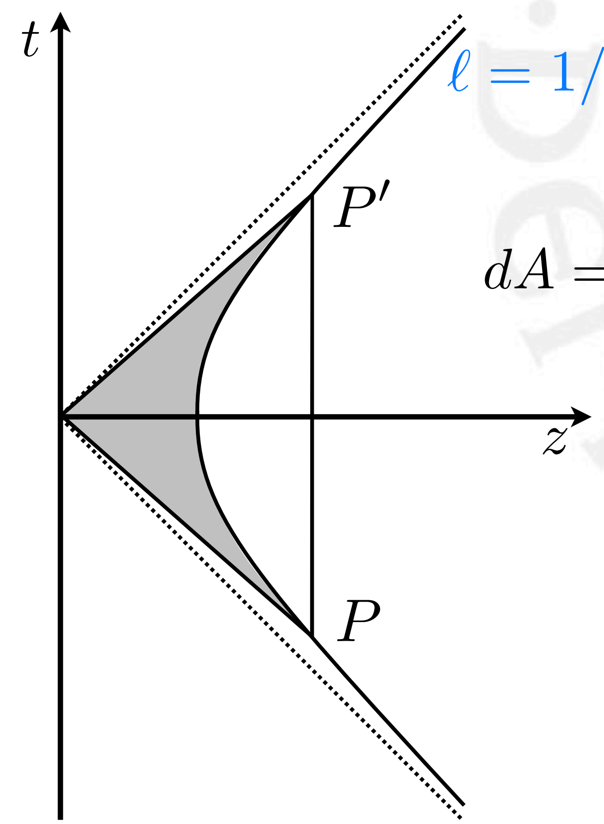
Spinfoam dynamics: $\vec{K} + \gamma \vec{L} = 0$

Gauge-fix the tetrads to be diagonal: **Lorentzian area** $A = \int_{\mathcal{R}} \gamma K^z = \int_{\mathcal{R}} L^z$

$$A_{min} = 4\pi G\hbar \qquad a_{max} = \sqrt{\frac{1}{8\pi G\hbar}} \qquad \ell_{max} = \sqrt{8\pi G\hbar}$$

- Wedge amplitude:

$$W(\eta, h) = \sum_j (2j+1)\ \mathrm{Tr}_j[Y^\dagger e^{i\eta K_z} Y h]$$
$$= \sum_j (2j+1)\ \mathrm{Tr}_j[e^{i\eta\gamma L_z} h]$$
$$= \sum_{j,m} (2j+1) e^{i\eta\gamma m} D^{(j)}(h)_{mm}$$



$\ell = 1/a$

$$dA = \frac{\ell^2}{2} d\eta = \frac{1}{2a^2} d\eta$$

- In the coherent-state basis :

$$W(\eta, j) = e^{i\eta 8\pi G\hbar\gamma j}$$

η is the boost parameter along the trajectory from P to P′

REMOVAL OF INFINITIES 1: singularities

- Minimal distance from the horizon: $\ell = R/\dot{R}$
- Maximal acceleration: $a \sim \sqrt{\ddot{R}/R}$
- Maximal energy density: $\rho_{\max} \sim \frac{3}{8\pi G} \frac{\dot{R}^2}{R^2}\Big|_{max} = \frac{3}{8\pi G}\ell_{min}^{-2} = \frac{3}{\hbar(8\pi G)^2}$
- **SPINFOAM**: singularity are avoided! [Rovelli, FV]

- Minimal volume - classically the conjugate variable is the Hubble rate
- **LQC**: holonomy corrections → bounded Hubble rate! effective eqs: $\ell_P\, \dot{R}/R \to \sin\left(\ell_P\, \dot{R}/R\right)$

- Strong singularity are solved: big bang, big crunch, big rip... [Singh, FV]
- Maximal acceleration: it may have implications for weak-singularity resolution [Rovelli, FV]
- No energy condition is violated. It is a pure quantum effect.

REMOVAL OF INFINITIES 2: finiteness of the amplitude

- Early perturbative quantum gravity: **non-renormalizability**
  - **Local:** *observables at arbitrarily small regions in a continuous manifold*
  - **Infinite** *renormalization group*
  - **Cut-off:** *it is a mathematical trick*
- Perturbations methods are some kind of approximation.
- Infinities: we perturb around points that are not really good.

- **Non-perturbative approach:** presence of a fundamental scale!
- Minimal area $a_o = 8\pi G\hbar\gamma\frac{\sqrt{3}}{2}$ $\rightarrow$ natural UV cut-off
- Cosmological constant $\Lambda > 0$ $\rightarrow$ natural IR cut-off
  - ↳ *horizon*

$$\phi_{min} = \sqrt{\Lambda}\,\ell_P$$

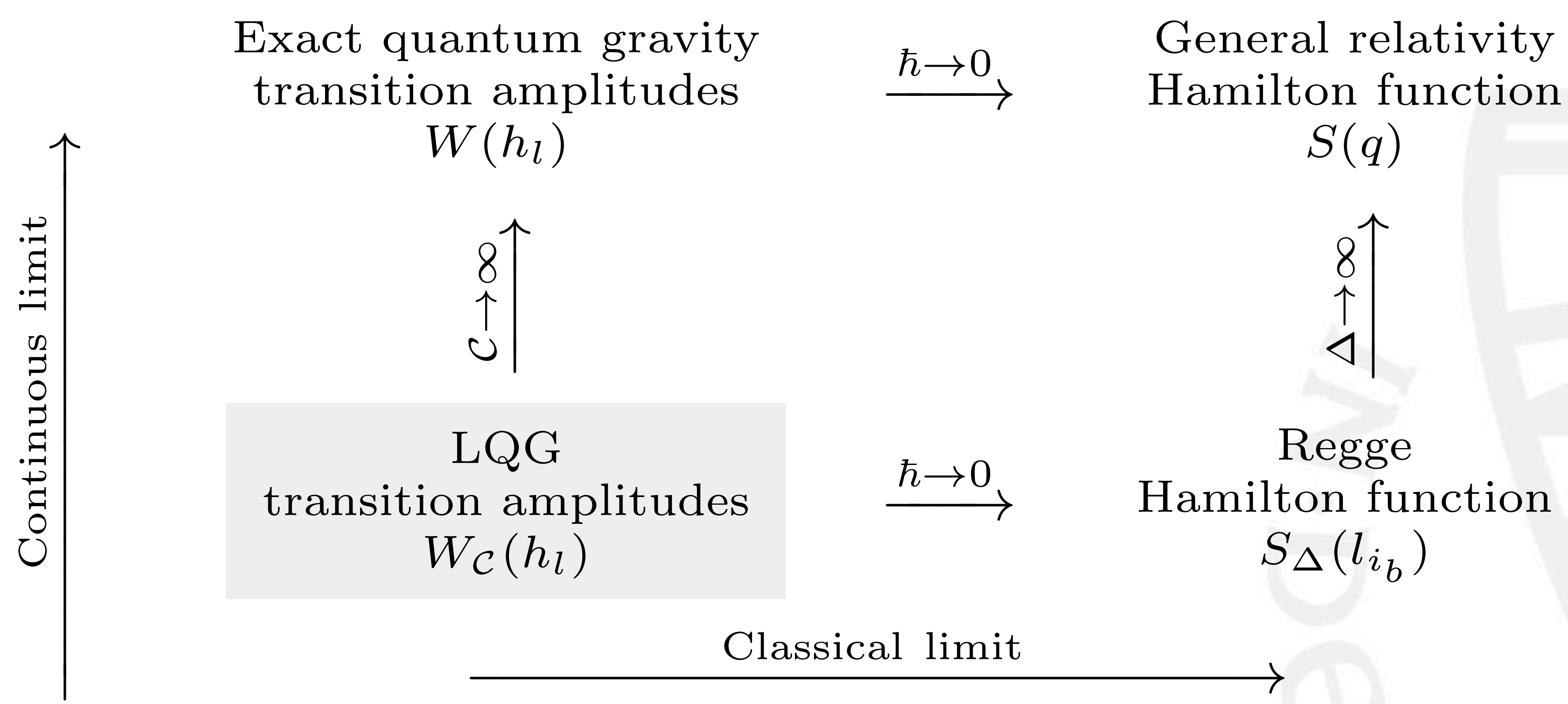
Han, Fairbairn, Moesburger, 2011
see also Bianchi, Rovelli 20111

- Planck length + horizon = minimal angular resolution j_{max}
- Mathematically a *fuzzy spheres:* spherical harmonics with $SU(2)_q$
- A maximum angular momentum characterizes the representations of $SU(2)_q$

$$q = e^{i2\pi/k} \qquad \text{with } k \sim 2 j_{max}$$

(Majid'88)

- The local rotational symmetry is better described by $SU(2)_q$ than by SU(2), with $q = e^{i\Lambda l_P^2}$
- Physically: non-commutativity, fuzziness of any angular function, impossibility of resolving small dihedral angles. (Connes'94)
- Loop gravity: ϕ is an operator with a discrete spectrum.
- Best angular resolution: $\phi_{min} = \sqrt{2/j_{max}}$ with $j_{max} \sim \frac{1}{l_P^2\,\Lambda}$ (Major'99)



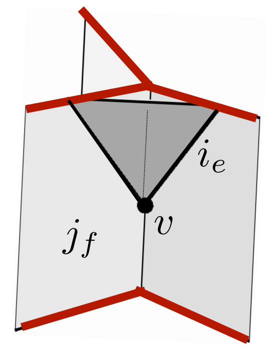
- No critical point
- No infinite renormalization
- Physical scale: Planck length

- Viability of the expansion:
  first radiative corrections are logarithmic (Riello)
- Regime of validity of the expansion:
  $L_{Planck} \ll L \ll \sqrt{1/R}$

prejudice 4

“quantum gravity
should have general relativity
as its classical limit”

LARGE DISTANCE LIMIT



Two-complex $\mathcal{C}$
(dual to a cellular decomposition)

$$Z = \sum_{j_f, i_e} \prod_f d_{j_f} \prod_v A(j_f, i_e)$$

Theorem :
[Barrett, Pereira, Hellmann, Gomes, Dowdall, Fairbairn 2010]

$$A(j_f, i_e) \underset{j \gg 1}{\sim} e^{iS_{\text{Regge}}} + e^{-iS_{\text{Regge}}}$$

[Freidel Conrady 2008, Bianchi, Satz 2006, Magliaro Perini, 2011]

$$W_{\mathcal{C}} \xrightarrow[j \gg 1]{} e^{iS_\Delta} \qquad Z_{\mathcal{C}} \xrightarrow[C \to \infty]{} \int Dg \;\; e^{iS[g]}$$

Theorem :
[Han 2012]

$$A^q(j_f, i_e) \underset{j \gg 1, q \sim 1}{\sim} e^{iS^\Lambda_{\text{Regge}}} + e^{-iS^\Lambda_{\text{Regge}}} \qquad q = e^{\Lambda \hbar G}$$

gravity

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$ds^2 = dt^2 - a^2(t)\, d^3\vec{x}$$

+ perturbations

canonical / covariant quantization

gravity → quantum gravity

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

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symmetry
reduction

cosmology

$$ds^2 = dt^2 - a^2(t)\, d^3\vec{x}$$

+ perturbations

quantum
cosmology

APPLICATIONS: SPINFOAM COSMOLOGY

canonical / covariant quantization

gravity

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quantum gravity

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symmetry reduction

cosmology

$$ds^2 = dt^2 - a^2(t)\, d^3\vec{x}$$

+ perturbations

quantum cosmology

$$H = const\,(a\dot{a}^2 - \frac{\Lambda}{3}a^3) = 0$$

$$\dot{a} = \pm\sqrt{\frac{\Lambda}{3}}a$$

$$G_\ell = G_{n_s} G_{n_t}^{-1}$$

1st-ORDER FARCTORIZATION

- classical dynamics

$$H = const\,(a\dot{a}^2 - \frac{\Lambda}{3}a^3) = 0$$

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1^{st}-ORDER FARCTORIZATION

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$$S_H = const \int \mathrm{d}t\,(a\dot{a}^2 + \frac{\Lambda}{3}a^3)\Big|_{\dot{a}\,=\,\pm\sqrt{\frac{\Lambda}{3}}a} = const\,\frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a_f^3 - a_i^3)$$

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- quantum dynamics

$$W(a_f, a_i) = e^{\frac{i}{\hbar} S_H(a_f, a_i)} = W(a_f)\overline{W(a_i)}$$

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- loop dynamics

$$\langle W | \psi_{H_{(z,z')}} \rangle = W(z, z') = W(z)\, \overline{W(z')}$$

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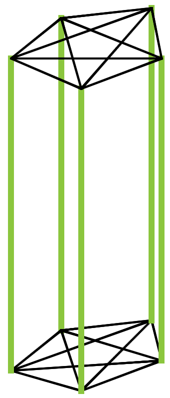
order (0)



$$= W_0(h_\ell, h_{\ell'}) = \delta_{\Gamma_\ell}(h_\ell, h_{\ell'})$$

$$G_\ell = G_{n_s} G_{n_t}^{-1}$$

1st-ORDER FARCTORIZATION

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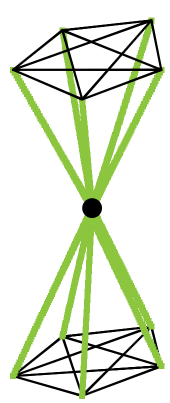
order (1)



$$W_{\mathcal{C}_\infty}(z', z) = \int h_\ell \int h'_\ell\ \overline{\psi_{z'}(h'_\ell)}\ W_1(h'_\ell, h_\ell)\ \psi_z(h'_\ell)$$

$$G_\ell = G_{n_s} G_{n_t}^{-1}$$

- classical dynamics

$$S_H = const \int \mathrm{d}t \, (a\dot{a}^2 + \frac{\Lambda}{3} a^3)\Big|_{\dot{a} = \pm\sqrt{\frac{\Lambda}{3}}a} = const \frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a_f^3 - a_i^3)$$

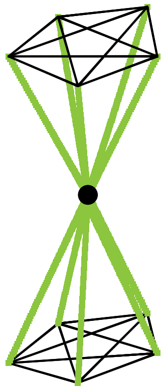
- quantum dynamics

$$W(a_f, a_i) = e^{\frac{i}{\hbar} S_H(a_f, a_i)} = W(a_f)\overline{W(a_i)}$$

- loop dynamics

$$\langle W | \psi_{H_{(z,z')}} \rangle = W(z, z') = W(z)\,\overline{W(z')}$$

order (1)



$$W_{\mathcal{C}_\infty}(z',z) = \int h_\ell \int h'_\ell \; \overline{\psi_{z'}(h'_\ell)}\; W_1(h'_\ell, h_\ell)\; \psi_z(h'_\ell)$$

$$W_1(h'_\ell, h_\ell) = \int_{SL(2,\mathbb{C})} \prod_{n=1}^{N-1} dG_n \prod_{\ell=1}^{L} P(h_\ell, G_\ell) P(h'_\ell, G'_\ell)$$

$$G_\ell = G_{n_s} G_{n_t}^{-1}$$

$$W(z) = \sum_{j_\ell} \prod_{\ell=1}^{L} \frac{1}{\alpha^3 \sqrt{\det \ Hess(j_\ell)}} (2j_\ell + 1)\, e^{-2t\hbar j_\ell (j_\ell+1) - izj_\ell} \ e^{-\frac{1}{2} i j_\ell \, \theta}$$

$$\theta\,(\gamma K + 1) - \theta = 0$$

- Gaussian sum peaked at j_o for all j_ℓ $\quad j \sim j_o + \delta j$
- max (real part of the exponent) gives where the gaussian is peaked; $\quad j_o \sim Im\,\tilde{z}/4t\hbar$
- imaginary part of the exponent =2kπ gives where the gaussian is not suppressed. $Re\,\tilde{z} = 0$ $\quad \dot{a} \sim 0$
- We obtain Minkowski space!

$$W(z) = \left(\sqrt{\frac{\pi}{t}}\, e^{-\frac{\tilde{z}^2}{8t\hbar}} \ 2j_o \right)^L \frac{N_\Gamma}{j_o^3}$$

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[Bianchi FV Krajewski Rovelli]

$$Z_{\mathcal{C}} = \sum_{j_f, \mathtt{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda\, \mathtt{v}_e} \prod_v A_v(j_f, \mathtt{v}_e)$$

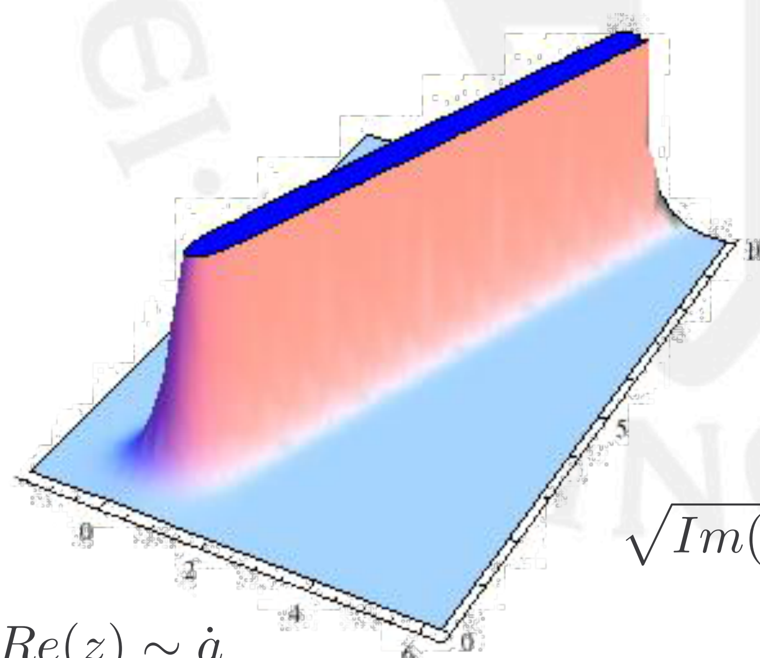
$$W(z) = \sum_j (2j+1)\, \frac{N_\Gamma}{j^3}\; e^{-2t\hbar j(j+1) - izj - i\lambda \mathtt{v}_o j^{\frac{3}{2}}}$$

$$\mathtt{v}_e \sim \mathtt{v}_o\, j^{3/2}$$

$$i\lambda \mathtt{v}_o j^{\frac{3}{2}} \sim i\lambda \mathtt{v}_o j_o^{\frac{3}{2}} + \frac{3}{2} i\lambda \mathtt{v}_o j_o^{\frac{1}{2}}\, \delta j$$

- the gaussian is peaked on $j_o = \dfrac{Im(z)}{4t\hbar}$
- the gaussian is not suppressed for $Re(z) + \lambda \mathtt{v}_o j^{\frac{1}{2}} = 0.$
- $\dfrac{Re(z)^2}{Im(z)} = \dfrac{\lambda^2 \mathtt{v}_o^2}{4t\hbar} \longrightarrow \left(\dfrac{\dot a}{a}\right)^2 = \dfrac{\Lambda}{3}$

$$\Lambda = const\, \lambda^2 G^2 \hbar^2$$



$\sqrt{Im(z)} \sim a$

$Re(z) \sim \dot a$

TO CONCLUDE

SUMMARY

- *"in quantum gravity spacetime should be quantized"*
  The theory predicts the existence of quanta of space.
  Minimal eigenvalue in the spectrum of geometrical quantities.
  Lorentz invariance is a basic ingredient, it is preserved.

- *"the theory should be so simple and short that it would fit on a tshirt"*
  The theory is defined by the triple: Hilbert space, observable algebra and transition amplitudes. All the objects are well defined.
  The amplitudes can be computed with cosmological states (SPINFOAM COSMOLOGY)

- *"quantum gravity should remove the infinities of general relativity"*
  The amplitudes are UV and IR finite: Planck length and Λ are fundamental.
  Renormalization: first radiative corrections are logarithmic.
  In cosmology, Spinfoam support and extend the resolution of cosmological singularities.

- *"quantum gravity should have general relativity as its classical limit"*
  The boundary states represent classical geometries.
  The classical limit of the vertex amplitude gives the Regge Hamilton function.
  In cosmology, Friedmann equations for Minkowski and deSitter are recovered.

Merci !