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Atomism and Relationalism as guiding principles for Quantum Gravity

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Abstract The research in quantum gravity has jauntily grown in the recent years, intersecting with conceptual and philosophical issues that have a long history. In this paper I analyze the conceptual basis on which Loop Quantum Gravity has grown, the way it deals with some classical problems of philosophy of science and the main methodological and philosophical assumptions on which it is based. In particular, I emphasize the importance that atomism (in the broadest sense) and relationalism have had in the construction of the theory.

Keywords Atomism · Relationalism · Quantum Gravity

Scientist are guided in their investigation of Nature by ideas brought to them by philosophers. Not always scientist are aware of this, but still their way of working and the kind of questions that they address, have emerged in a humus of philosophical debates of long history. Being aware of these debates, that shaped our present investigations, can help us to recognize paths and new directions for our science.

I think this is particularly true when the theory we are searching for requires a deep rethinking of basic concepts such as space and time. In order to analyze the recent findings in quantum gravity, a theory that hopes to provide a new fundamental view on the nature of space and time, I start by a possible reconstruction of the thread from the ancient time to our modern debate.

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1 The historical framework: from Democritus to Einstein

What does exist? Does space exists? or it emerges from the relations between *bodies*? The idea that space can exist as a separate entity with respect to the bodies is not a primitive idea, but a revolution brought to humanity by Leucippus and Democritus in the V century BCE. They postulates the existence of elementary units, the *atoms*, whose combination yields all the beautiful rainbow of different things that we observe in the world. The atoms randomly moves to a stage: this is the space. Space is here associated with the notion of vacuum. This is a contradictory notion, because gives an ontological status to the non-being. That's why this position was attached by Athens' school: Aristotele thought that Nature *abhors a vacuum*. Plato did not like the atomistic/materialistic views either, at the point never to mention Democritus. Trough Greek decadence and the Cristian era, only Atheniensis wisdom survived to the centuries.

The XVII-century debate on the nature of space should be framed in a culture dominated by the aristotelian/platonic thinking. The mainstream side of the debate was there taken by the *relational* position, defended by Descartes and Leibniz, denying the existence of space but as a net of relations. On the other side there was the substantivalist position proposed by Newton, in which democritean bodies moves on an infinite immutable fixed empty space, according to the deterministic law of the new Mechanics. The empirical success of Mechanics has brought this position to became the mainstream one nowadays.

Nonetheless, Newton hesitated in proposing such an idea of space. He called this a working hypothesis (*hypotheses non fingo*) and its enormous success as funding stone of the new developing science made the later scientists forget the doubts about it. What concerned Newton more was his law of gravitation, acting on an empty stage, were leading to the possibility of an *action at distance*. Only with the introduction of the notion of field this worry was removed: forces are fields, that permeate space.

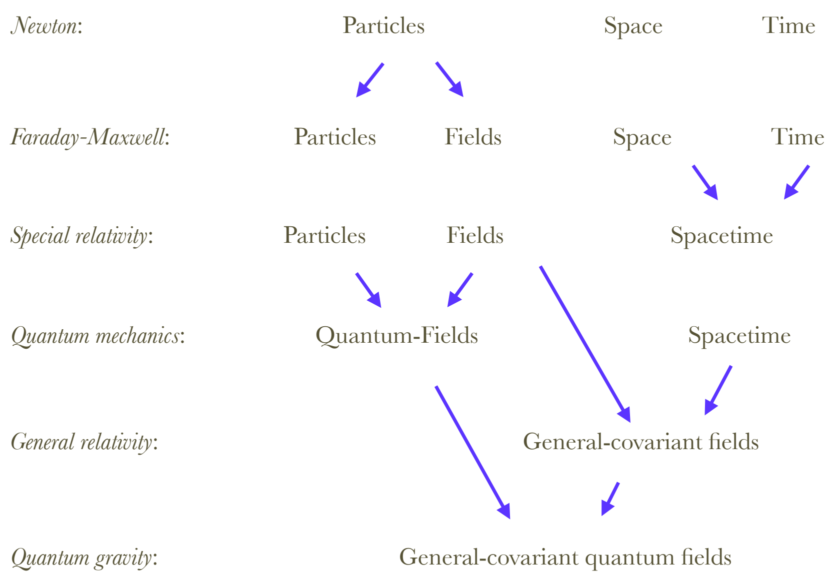
Faraday and Maxwell described the electromagnetic phenomena as a manifestation of a field. The physics of the XX century took the notion of field to an ontological extreme: everything that exist is a manifestation of some field. So it is every particle, as discovered by quantum mechanics: a particle is just the excitation of a field, a manifestation of the quantum nature of every field. So it is space, as discovered by Einstein: space and time are the expression of the gravitational field.

General relativity identifies spacetime and the gravitational field. This field, like all fields, exhibits quantum properties at some scale, therefore space and time must have quantum properties as well. This is the beauty and the difficulty of quantum gravity: it obliges us to a complete rethinking of what we mean by space and time. In order to do this, we need to sharp our description of quantum fields in order to make it covariant (i.e. compatible with general relativity). We have to learn a new language for describing the world. A language which is neither that of standard field theory on flat spacetime, nor that of Riemannian continuous geometry as space present the discreteness typical of every quantum systems. We have to understand what is quantum space and what is quantum time.

2 The end of space and time, the beginning of quantum gravity

As often happens in science, the contemporary questioning of the nature of space started with a mistake. The problem was to extend Heisenberg's uncertainty relations to fields. In a 1931 Landau and Peierls suggested that once applied to fields, the uncertainty relation would imply that no component of a field at a given spacetime point could be measured with arbitrary precision [1]. The intuition was that an arbitrarily sharp spatiotemporal localization would have been in contradiction with the Heisenberg uncertainty relations. Niels Bohr guessed immediately, and correctly, that this suggestion was wrong. To prove it wrong, he embarked in a research program with Rosenfeld, that lead to a classic paper [2] in which the two proved that in a quantum field theory the Heisenberg uncertainty relations *do not* prevent a component of a field to be measured with arbitrary precision at a spacetime point. The Bohr-Rosenfeld analysis was done using the electromagnetic field: what if repeated with the gravitational field? This question engaged Landau's his friend Bronstein [3] and he found that Landau's intuition in this case was correct [4, 5]. This is the beginning and the core of quantum gravity.

In modern terms, Bronstein's argument would be the following. In order to measure some field value at a location x, its location should be determined, say with a precision L. If this is done by having a particle at x, the quantum nature of the particle particle implies that there is an uncertainties



Δx and Δp associated to its position and its momentum. To have the location determined with precision L, this should be greater than Δx, and since Heisenberg uncertainty gives $\Delta x > \hbar/\Delta p$, we have $\Delta p > \hbar/L$. The average absolute value of the momentum cannot be smaller than its fluctuation, therefore $|p| > \hbar/L$. This is a very well known consequence of Heisenberg uncertainty: sharp location requires large momentum. In turn, large momentum implies large energy E. In the relativistic limit, where rest mass is negligible, $E \sim cp$. Sharp localization requires large energy.

In general relativity any form of energy E acts as a gravitational mass $M \sim E/c^2$ and distorts spacetime around itself. The distortion increases when energy is concentrated, to the point that a black hole forms when a mass M is concentrated in a sphere of radius $R \sim GM/c^2$, where G is the Newton constant. If L arbitrary small in order to get a sharper localization, the concentrated energy will grow to the point where R becomes larger than L. But in this case the region of size L that we wanted to mark will be hidden beyond a black hole horizon, and we loose localization. Therefore L can be decreased only up to a minimum value, which clearly is reached when the horizon radius reaches L, that is when $R = L$.

Combining the relations above, we obtain that the minimal size where we can localize a quantum particle without having it hidden by its own horizon, is

$$L = \frac{MG}{c^2} = \frac{EG}{c^4} = \frac{pG}{c^3} = \frac{\hbar G}{Lc^3}. \tag{1}$$

Solving this for L, we find that it is not possible to localize anything with a precision better than the length

$$l_o \ \sim \ \sqrt{\frac{\hbar G}{c^3}} \ \sim \ 10^{-33} \ cm, \tag{2}$$

which is called the Planck scale. Above this length scale, we can treat spacetime as a smooth space. Below this scale, it make no sense to talk about distance or extension.

This simple derivation, using only semiclassical physics, characterizes the physics of quantum spacetime. The existence of a minimal length scale is the main feature of quantum gravity and gives it a universal character, analogous to special relativity and quantum mechanics. Special relativity can be seen as the discovery of the existence of a maximal local physical velocity, the speed of light c. Quantum mechanics can be interpreted as the discovery of a minimal action $\hbar$ in all physical interactions, or equivalently the fact that a finite region of phase space contains only a finite number of distinguishable (orthogonal) quantum states, and therefore there is a minimal amount of information in the state of a system. Quantum gravity is the discovery that there is a minimal length.

In Bronstein's words [4]: "Without a deep revision of classical notions it seems hardly possible to extend the quantum theory of gravity also to [the short-distance] domain." Bronstein's result forces us to take seriously the connection between gravity and geometry. It shows that the Bohr-Rosenfeld argument, showing that quantum fields can be defined in arbitrary small regions of space, fails in the presence of gravity. Therefore the quantum gravitational field cannot be treated simply as a quantum field in space. The smooth metric geometry of physical space, which is the ground needed to define standard quantum field, is itself affected by quantum theory. What we need is a genuine quantum theory of geometry.

3 Quanta of spacetime

The lesson of General Relativity yields background independence: spacetime is not the playground for field interactions, but is an interacting field as the others. Our modern understanding of fields is that they are associated to some gauge symmetry. In the Standard Model, these symmetries are respectively $U(1)$ for electromagnetism, $SU(2)$ for the weak interaction and $SU(3)$ for the strong interaction. Gravity does not differ and can be written as a gauge theory.

General Relativity admits a version where, instead of the metric, the fundamental object are reference fields: the tetrads. One can always express the metric in terms of the tetrads, therefore the tetrad version can be consider equivalent to the original metric formulation. On the other hand, only in the tetrad formulation it is possible to couple fermions to the gravitational field: since we observe fermions in nature, this makes the tetrad version of General Relativity somehow more fundamental.

So let us start by the tetrads. The invariance under diffeomorphisms of General Relativity imposes the independence by coordinate transformations. This implies that for each point of spacetime we have to associate a tetrad that is locally Lorentz invariant. But time is pure gauge in General Relativity and we can always fix this gauge, leaving us only with the rotational part of the Lorentz transformation. This gauge invariance, that naturally arises in the classical gravitational theory, is the starting point in order to jump to the quantum theory. The quantum states have to be thought as boundary states [6,7], describing the space geometry at some fixed time. When it comes to quantization, the tetrad turns out to be the generator of SU(2) transformations, satisfy the well-known algebra of the angular momentum. This implies that spacetime is quantized with a discrete spectrum. So actually we don't have any more a tetrad for each spacetime point, but a tetrad for each quanta of spacetime. On each quanta of spacetime it does not matter how the reference fields are oriented, but only the relations between adjacent quanta. A spinnetwork state in Loop Quantum Gravity [8,9] is a gauge invariant state (invariant under the rotations of the triads) that knows about the excitations of each quanta of spacetime (its spin, that is related to its physical size) and the adjacency relation between them (coded in an abstract graph).

The gauge invariance of the triads yields the presence of a gauge field, as in the other Yang-Mills theories. This is an object in the Lorentz algebra, that codes the information about (intrinsic and extrinsic) curvature. In order to define gauge invariant observables in the quantum theory, we consider the path-ordered exponential of the gauge field. This is called Wilson loops in the language of particle physics (from this the designation Loop for the quantum theory) or holonomy in the language of differential geometry: if the gauge field is seen as the connection over the $SU(2)$ principal bundle, the holonomy is its parallel transport. This turns out to be the canonically conjugate variable to the triad.

Loop Quantum Gravity variables are group variables, as the variables of the other interactions are. Since $SU(2)$ is a compact group, the spectrum of the observables corresponding to these group variables are discrete. In particular, in Loop Quantum Gravity the geometry can be describes trough observables such as areas, volumes and angles, constructed starting from the operator corresponding to the triads.

4 Quantum relations

We want now to discuss the dynamical properties of spacetime. In classical mechanics, a dynamical system can be defined trough the relations between initial and final coordinates and momenta, defining the allowed trajectories. In quantum mechanics, the trajectories between interactions are not deducible from the interaction outcomes, and the theory describes *processes*. The quantum analogs of the relations between values of physical variables at the *boundaries* are the transition amplitudes W, which determine probabilities of alternative sets of outcomes.

Quantum mechanics describes the manner physical systems affect one another in the course of interactions [10]. It computes the probabilities for the different possible effects of such interactions. A common language for describing such processes is in terms of "preparation" and "measurement". But this anthropomorphic language is misleading [10]. What happens at the boundary of a process if simply a physical *interaction* of the system with another –completely generic– physical system. Notice that the structure of the theory is largely determined by the fact that this description is consistent with arbitrary displacements of what we decide to consider as the boundaries between processes.

Let us now apply this perspective to field theory and in particular to gravity. The central idea is now to consider a *finite* portion of the trajectories of a system, where "finite" means finite in time but also in space (this is called the "boundary formalism" [6,7]). Thus, we focus on a a finite bounded region M of spacetime. For a field theory on a given fixed spacetime, this simply means that we consider the evolution of the field in a spacial box, with given boundary values at the boundaries of the box. Therefore the transition amplitudes will be functions of the field values on the initial spacelike surface, the final spacelike surface, but also on the "sides": the timelike surface that bounds the box. In other words, the transition amplitudes W are functions of the values of the field on the entire boundary of the spacetime region M. Formally, W can be expressed as the Feynman path integral of the field in M, with fixed values on the boundary $\Sigma = \partial M$. Obviously, W will depend on these values of the fields as well as on the (spacetime) shape and geometry of Σ. For instance, on the time lapsed between the initial and final surfaces.

Now let us take this same idea to gravity. Then what we want is the transition amplitude W that depend on the value of the gravitational field (as well as any other field which is present) on the boundary Σ of a spacetime region. Formally, this will be given by the Feynman path integral in the internal region, at fixed boundary values of the gravitational (and other) fields Σ. How do we now specify the shape, namely the geometry, of Σ? Here come the magic of quantum gravity: the answer is that we do not.

In fact, the gravitational field on the boundary of Σ *is* already specifies the shape of Σ! It includes any relevant metric information that can be gathered on the surface itself! Therefore we expect that W will be a function of the boundary fields *and nothing else*.

This happens as a consequence of a general property of parametrized systems: the temporal information is stored and mixed among the dynamical variables, instead as being signed out and separated from other variables as in unparametrized Newtonian mechanics. In the general relativistic context, this holds for temporal as well as for spacial locations: W will not be a function of space and time variables, but simply a function of the gravitational field on the boundary Σ (up to diffeomorphisms of Σ), which contains the entire relevant geometrical information on the boundary.

Therefore in quantum gravity the quantum dynamics will be captured by a transition amplitude W which is a function of the (quantum) state of the field on a surface Σ. Intuitively, W is the “sum over geometries” on a *finite* bulk region bounded by Σ: this is called a spinfoam. The explicit form of W is one of the most important result in quantum gravity of the last years [11–16].

5 The relational structure of quantum gravity

The formal structure of quantum mechanics is relational, because quantum mechanics gives probability amplitudes for processes, and a process is what happens between interactions [10]. Therefore quantum mechanics describes the universe in terms of the way systems affect one another. States are descriptions of manners a system can affect another system. Quantum mechanics is therefore based on *relations* between systems, where the relation is instantiated by a physical interaction.

The structure of general relativity is also relational, because the localization of dynamical objects is not given with respect to a fixed background structure; instead, *bodies* are only localized with respect to one another, where *bodies* includes all dynamical objects, included the gravitational field. The relevant relation that build the spacetime structure is of course contiguity: the fact of being “next to one another” in spacetime. We can view a general relativistic theory as a dynamical patchwork of spacetime regions adjacent to one another at their boundaries.

A fundamental ingredient in XX century physics is locality: interaction are local, namely they require spacetime contiguity. But the contrary is also true: the only way to ascertain that two objects are contiguous is by means of having them interact. Therefore locality reveals a fundamental structural analogy between the relations on which quantum mechanics is based and those on which spacetime is based. Quantum gravity makes this connection completely explicit: a process is not in a spacetime region: a process *is* a spacetime region. A state is not somewhere in space: it *is* the description of the way two processes interact, or two spacetime regions pass information to one another. Viceversa, a spacetime region *is* a process: because it is actually like a Feynman sum of everything can happen between its boundaries:

Quantum Mechanics		General relativity
Process	$\leftarrow$ Locality $\rightarrow$	Spacetime region
State		Boundary, space region

This structural identification is in fact much deeper. As noticed, the most remarkable aspect of quantum theory is that the boundary between processes can be moved at wish. Final total amplitudes are not affected by displacing the boundary between "observed system" and "observing system". The same is true for spacetime: boundaries are arbitrarily drawn in spacetime. The physical theory is therefore a description of how arbitrary partitions of nature affect one another. because of locality and because of gravity, these partitions are at the same time subsystems split and partitions of spacetime. A spacetime is a process, a state is what happens at its boundary.

6 Conclusion

What does exist? Does space exists? or it emerges from the relations between *bodies*? Quantum gravity is the quest for a synthesis between quantum mechanics and general relativity. But while doing this, quantum gravity would achieve a synthesis also between substantivalism and relationalism: space is a field, that come to existence only trough its interactions. Space is constitute by atoms of space, defined trough their relations.

Appendix: Against Infinity

It is usually said that a quantum theory of gravity should remove the infinities that appear in general relativity in the form of singularities. Such a statement reveals a distinct attitude towards infinities: they are features of our limited knowledge, not an existing property of Nature. Such an approach to infinities is present in Loop Quantum Gravity, in different ways. Three kind of infinities are cured: I describe them briefly.

A.1 Observables

There is a diffused prejudice according to which in quantum gravity only observables at infinity are to be considered. This prejudice has been nourished by numerous historical sources, but it is misleading. One source is simply particle theory, with its focus on scattering, where observables at infinity are particularly convenient. A second source is the difficulty of defining bulk observables in a generally covariant theory. This difficulty is real, but it can be resolved. We are capable of describing the dynamics of our Solar System and the detailed measurements on it, in fully general relativistic terms, and well into the general relativistic regime, in spite of the fact that we are immersed in the Solar System and certainly are not at infinite distance from it. Therefore obviously there exist well defined observations and observables which are general covariant and are not at infinity: difficult does not mean impossible. The distinction between partial observables and full observables is the basis for a simple solution of this difficulty. A third source for the prejudice is the idea that local observable require infinite precision and this can only be achieved with infinitely long or infinitely extended measurements. In particular, in a region of size L it does not make sense to discuss time evolution with a time resolution better than $\delta t \sim (L_0^2/L)$ (see [17]). The mistake in this argument is that it *presumes* a background continuous spacetime with respect to which t is measured, which is exactly what is not anymore there in quantum gravity. It is of course true that time resolution is limited by uncertainty relations, but this is consistent with standard uncertainly relations on the gravitational field at the boundary. It is spacetime (as determined by the gravitational field) that becomes fuzzy at the boundary, and this perfectly taken into account by quantum gravity, on *any* boundary. We can make any measurement in any place: what is fuzzy is the measured value of the gravitational field, therefore the physical localization of the measurement with respect to something else, and not the possibility itself of making a measurement somewhere else than infinity. A fourth source for the prejudice is the set of observations and ideas that cluster around the name of holography (see again [17]). The idea is that because of the Bekenenstein bound the number of states of any apparatus is bounded by the area around it, therefore an apparatus localized in spacetime can only distinguish a finite number of states and therefore cannot resolve arbitrary small distances. This again is correct, and in fact, it is precisely what is captured by the physical granularity of spacetime. All these arguments consistently show that in the presence of gravity there are no local observables in the sense of local quantum field theory: that is, localized in arbitrary small regions. In other words, all these arguments are reruns of Bronstein's original argument on the fact that space and time are ill defined in quantum gravity. But the solution is not to take refuge at infinity. The solution is to accept observables that do not resolve space and time more finely than Planck scale. Because there is physically no space and time at scales finer than Planck scale. Is conventional quantum field theory that should be upgraded, to be able to deal with observables that are local in a more general sense. This is precisely what the boundary formalism, which is compatible with a fuzzy geometry, does. There is no need to use observables at infinity for quantum gravity.

A.2 Singularities

The presence of a minimal eigenvalue in the discrete spectrum of the area plays in cosmology the same role of the minimal eigenvalue of the angular momentum for atomic physics. In the classical theory, all the trajectories fall into a singularity: the electrons spiral down into the atomic nucleus, all the matter of the universe gets evolved back in time into the big bang. The dissolution of continuous spacetime into quanta of spacetime prevents singularity theorems to apply. Singularity theorems do not point to the presence of some infinity in Nature, but they rather signal the boundary beyond which the classical theory ceases to be valid.

The Loop quantization removes all the cosmological (physical) singularities [20–22]. It is a genuine consequence of the quantization, rather than the result of some exotic condition imposed to our universe. The result has proven to hold even in presence of anisotropies and inhomogeneities. There is no fine-tuning of initial conditions, nor an ad hoc boundary condition at the singularity. Furthermore, matter can satisfy all the standard energy conditions. Notice that there exist a zoo of possible cosmological singularities, beyond the notorious big bang: big rip singularities, sudden singularities, big freeze singularities, big brake... When the singularity regards spacetime itself, taking the form of a divergence in the curvature or of its derivative, loop quantization promptly resolve it. If instead the singularity is a divergence in the pressure or its derivative, loop quantization seems to have nothing to say: these are not singularity where spacetime breaks as geodetics can be continued trough them [23–26].

A.3 Renormalization

Quantum discreteness of spacetime is a powerful achievement, but it is only the starting point of a beautiful journey into a new quantum theory. The kinematical space of Loop Quantum Gravity has some powerful elements of novelty, with deep consequence for our understanding of space and time through cosmological abyss. It opens the door to a new world, where new infinities must to be faced in order for the theory to survive.

The early attempts to quantize gravity with perturbative techniques got stuck because of the non-renormalizability of the resulting theory. The non-renormalizability obtained in perturbation theory has pushed the theoreticians into a quest for a larger renormalizable or finite theory. The quest has wandered through the investigation of modifications of GR with curvature square terms in the action, Kaluza-Klein-like theories, supergravity, and has lead to String Theory, a presumably finite quantum theory of all interactions including gravity, defined in 10 dimensions, including supersymmetry and so far difficult to reconcile with the observed world.

In a non-perturbative approach to quantum gravity, this problem can be overcome having clear in mind that the existence of the Planck length sets quantum gravity aside from standard quantum field theory for two reasons. First, we cannot expect quantum gravity to be described by a local quantum field theory, in the strict sense of this term [27]. Local quantum theory requires quantum fields to be described by observables at arbitrarily small regions in a continuous manifold. This is not going to happen in quantum gravity. Second, the quantum field theories of the standard model are defined in terms of an *infinite* renormalization group. The existence of the Planck length indicates that this is not going to be the case for quantum gravity.

When computing transition amplitudes for a field theory using perturbation methods, infinite quantities appear. What is the nature of these infinities? Perturbation methods are some kind of approximation. Infinities arise because we perturb around points that are not really good. In Feynman graphs the interaction for small loops take us to an arbitrarily small dimension, but this is not a physical fact: it is a mistake of the approximation used. Renormalization is a powerful tool for the computation, but it does not mean that every time that there is a phenomenon at some scale there are all the infinities down of that scale. In fact, renormalization theory is exactly what tells us that we can correct all that just by readjusting the coefficients. Even in a renormalizable Quantum Field Theory, where we need to compute only a finite number of (properly chosen) degrees of freedom, renormalization has to be used.

The standard technique consists in the introduction of a cut-off which removes the infinities. The definition of the theory is adjusted so that the cut-off dependence tune the physical observables to match the experimental observations. The cut-off can be regarded as a technical trick, not something physical. Accordingly, care is taken so that the final amplitudes do not depend on the short scale cut-off. This general structure has proven effective for describing particle physics, but it is not likely to be the structure that works for quantum gravity.

In quantum gravity the cut off in the modes is not a mathematical trick for removing infinities, but a genuine physical feature of the quantum spacetime. The Planck length provides an ultraviolet physical cut off. The cosmological constant provide an infrared cut off. These make the theory finite.

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